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Analytical formulas for estimation of phase-averaged parameters of random waves

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ABSTRACT

New analytical formulas for the estimation of wave radiation stress and wave mass flux are developed in this study based on asymptotic analysis of linear wave theory with the aid of non-directional parameterized wave spectra. Verification against synthetic wave data shows that the new formulas can allow up to 10% more accurate results compared to the use of the traditional representative wave approach. Sensitivity analysis based on reliable wave buoy data reveals that the performance of the new formulas may vary by up to 20% depending on the relative wave height, wave steepness, and spectral width. For a common sea state dominated by wind-induced waves, the formulas are most sensitive to the relative water depth $k_p h$ and their best applicable range is in upper transitional to deep water conditions ($k_p h \geq \pi/2$). The formulas were finally validated against nine independent sets of field wave spectra to reassure their superiority over the representative wave approach. The validation confirms that the new formulas can offer up to 10% and 15% improvements in terms of the estimation accuracy and the estimation precision, respectively.

1. Introduction

The balance of momentum and fluid mass over a series of water waves is typically described in terms of wave radiation stress and wave mass flux. These important wave parameters allow an explanation for many essential phenomena in the sea, including wave setup and setdown, undertow, longshore current, edge waves, and shear waves (e.g. Svendsen, 2006; Davidson-Arnott, 2010). An effective means to determine the parameters is required at a wide range of water depths. Accurate offshore estimates of the parameters are crucial for the preparation of wave climatology data which serve as numerical model inputs. These wave forcing terms are also important in the description of wave transformation towards the shoreline (e.g. Hughes, 2004; Wargula et al., 2013). The capability to estimate the wave radiation stress and wave mass flux therefore plays a very important role in many types of ocean circulation and coastal morphodynamic models (e.g. DeVriend and Stive, 1987; Haas et al., 2003; Srisuwan and Work, 2015).

In a phase-averaged sense, the radiation stress and wave mass flux are estimated per their total quantities over the wave period. Some advanced wave theories may permit the estimation on a phase-resolving, wave-by-wave basis (e.g. Rakha, 1998; Li et al., 2007). The intra-wave approach may enable a direct simulation of non-linear wave phenomena, but sometimes dynamic processes that involve uncertainty and randomness still cannot be described clearly enough for its practical implementations. The use of the phase-averaged estimation has been attempted in many research efforts to show that such an approach can lead to satisfactory modeling results under a wide range of wave conditions (e.g. Ding et al., 2006; Haas and Warner, 2009; Cambazoglu and Haas, 2011).

For monochromatic sinusoidal waves, linear wave theory may be applied for an estimation of the wave mass flux and wave radiation stress, following the original concept suggested by Longuet-Higgins and Stewart (1964). In a random wave field, two typical means for the estimation are found in current practice (e.g. Dean and Dalrymple, 1991; Holthuijsen, 2007). A numerical integration may be performed on a full surface wave energy spectrum obtained via measurement or parametrization. This method accounts for incremental contributions from all individual waves considering their frequency-dependent magnitudes and kinematics. The other simpler means to approximate the parameters is often referred to as a representative wave approach in which the estimation of the parameters is based solely on some nominal waves under an assumption of a narrow-banded wave field.

Improvements in the estimation of the wave radiation stress and the wave mass flux are found in the literature, most of which are achieved by use of extended wave theory for describing the wave

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Nomenclature

$S_{\alpha\beta}$	wave-radiation stress in α and β components	FL^{-1}
h	mean water depth	L
h_o	still water depth	L
$\bar{\eta}$	wave setup level	L
u_w	wave-induced water particle velocities	LT^{-1}
η	instantaneous water surface level	L
M_α	wave-induced mass flux in α component	MT
S_{xx}	wave-radiation stress in normal x direction	FL^{-1}
M_x	wave-induced mass flux in x direction	MT
E	directional surface wave energy spectrum	L^2T
C	wave celerity	LT^{-1}
C_g	wave group celerity	LT^{-1}
f	wave frequency	T^{-1}
H	surface wave height	L
H_{rms}	root-mean-square height of random waves	L
F_η	nondirectional surface wave energy spectrum	L^2T
D	directional spreading function of wave spectrum	[-]
ϑ	coefficient in parametrized wave spectrum	[-]
f_P	peak wave frequency	T^{-1}
γ^δ	peak enhancement factor in JONSWAP spectrum	[-]
ϕ_k	depth-dependency factor in TMA spectrum	[-]
k	wave number	L^{-1}
k_P	wave number evaluated at peak wave frequency	L^{-1}
f_L	low frequency cutoff	T^{-1}
f_H	high frequency cutoff	T^{-1}
ω	wave angular frequency	T^{-1}
θ	incident wave angle	[-]
u	nondimensional parameter equal to $(\omega^2 h)/g$	[-]
u_L	nondimensional parameter u evaluated at f_L	[-]
u_H	nondimensional parameter u evaluated at f_H	[-]
L	wave length	L
L_o	deep-water wave length	L
R^2	coefficient of determination	[-]
Err.	error in estimation as percentage of mean measured value	%
Std.	standard deviation of Err. as percentage of mean measured value	%
RMSD	root-mean-square deviation as percentage of mean measured value	%
BIAS	slope of a linear line fitted through comparison between two datasets	[-]

quantities and their variations. For inside and outside of the surf zone on gently sloping beaches, Stive and Wind (1982) applied two nonlinear wave theories to determine the radiation stress and wave setup and showed that they both are qualitatively superior to the linear wave theory. Wang et al. (2008) introduced new expressions of the wave radiation stress and volume flux, reformulated by including up to the sixth-order description of surface waves. The new formulas were incorporated in a circulation model which was tested against sinusoidal waves in intermediate to deep water depths. The formulas were able to produce an accurate estimation for wave setup and setdown with, however, a very marginal improvement on the velocity profile of wave-induced currents.

More practical improvements in the subject area have been focused on estimating the parameters for a group of random waves. Such efforts are partly motivated by the fact that a narrow-band representation of the waves may cause the resulting parameters to deviate significantly from their actual quantities in the sea (Tayfun, 1986). In this type of study, the parameters obtained via a full numerical integration of the random wave spectra are typically treated as the factual best estimates, to which the estimation results yielded by any other techniques are compared and evaluated. Feddersen (2004) attempted to investigate the validity of the narrow-band approximation on a set of locally-generated waves under intermediate to deep water conditions with a wide range of wave steepness. The study showed that the approximation could lead to around 35% overestimation in the radiation stress. The selection of waves to serve as the representative parameters alone could lead to much inconsistency in the estimation.

In the present study, two novel formulas are derived and introduced for the estimation of the total wave mass flux and wave radiation stress in the normal direction. While appearing in closed form and ready for analytical applications, the new formulas offer enhanced estimation accuracies, surpassing those allowed by the representative wave approach. The derivation of the formulas starts in the next section after a brief review of related wave theory. The new solutions are then verified against synthetic time series of random waves and their best application ranges are disclosed. Various sets of field wave data are employed in a further validation, including a sensitivity analysis, in order to ensure the enhancement in the estimation capabilities. Conclusions of the study are finally made to summarize the underlying bases and practical utilities of the new formulas.

2. Estimation of wave radiation stress and wave mass flux

The underlying physics of the wave radiation stress and wave mass flux will be described here along with a review of existing techniques for estimating the parameters. The new formulas will be compared to these typical techniques later on.

2.1. Spectral-based and narrow-banded approximations

A definition sketch is given in Fig. 1 for important wave parameters that contribute to the momentum and mass fluxes under a progressive wave of height H propagating over a mean water depth h. The radiation stress is defined as the excess flow of momentum due to the presence of the wave (Longuet-Higgins and Stewart, 1964). The term can be derived using a momentum balance equation, and is fully expressed as

$$S_{\alpha\beta} = \overline{\int_{-h_o}^{\eta} [\rho u_{w\alpha} u_{w\beta} + \delta_{\alpha\beta} P]\, dz} - \delta_{\alpha\beta}\left[\frac{1}{2}\rho g \overline{\eta^2}\right] \quad (1)$$

with $S_{\alpha\beta}$ being the 2nd order tensor of the radiation stress in the horizontal directions α and β. The factor u_w indicates the fluid particle velocity induced by the wave; P is the combined dynamic and hydrostatic pressure; and δ is the Kronecker delta. The coordinate and other referencing symbols, including η, $\bar{\eta}$, and h_o, are as defined in Fig. 1. Similarly, the wave mass flux is defined as the rate of fluid mass flow passing through the vertical plane, given as follows:

$$M_\alpha = \overline{\int_{-h_o}^{\eta} [\rho u_{w\alpha}]\, dz} \quad (2)$$

in which M_α is a per-unit-width mass flux directed in the α direction. In

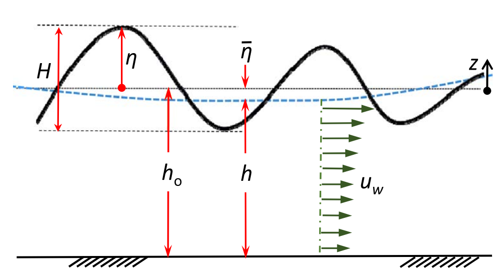

Fig. 1. Definition sketch of wave parameters associated with momentum and mass fluxes through a vertical plane.