Ocean Engineering 133 (2017) 66–72

Contents lists available at ScienceDirect

Ocean Engineering

journal homepage: www.elsevier.com/locate/oceaneng

Analytical models of sub-supercritical ship hydrodynamic pressure field with the dispersive effect

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ARTICLE INFO

Keywords:
Restricted waterways
Dredged channel
Subcritical speed
Supercritical speed
Sub-supercritical mixed flow
Ship hydrodynamic pressure field (SHPF)

ABSTRACT

Based on the potential flow theory, taking the dispersive effect into account, a shallow-water wave equation which satisfies Laplace equation, free surface and seabed boundary conditions is established. According to the slender ship assumption and the continuous matched condition on the interface of inner and outer regions, the mathematical problems of sub-supercritical mixed flow are analytically solved by using the Fourier integral transform method, meanwhile, the analytical models of sub-supercritical ship hydrodynamic pressure field (SHPF) in dredged channel are derived, and those of open water, rectangular canal and stepped canal can also be obtained by further simplifying or adopting similar method. The distribution characteristics of sub-supercritical SHPF in dredged channel are acquired, and the effects of transverse distance, inner or outer water depth, width, and depth Froude number on SHPF are analyzed. The SHPF analytical models with the dispersive effect are verified by comparing with the corresponding experimental results.

1. Introduction

The characteristic of ship wave in shallow water is closely related to the depth Froude number $F_h = V/\sqrt{gh}$, here V is the ship speed, h is the water depth, g is the gravitational acceleration, $F_h < 1$ and $F_h > 1$ are called as the subcritical and supercritical speed respectively. If a ship moves in shallow water with constant depth, the flow may only exist one depth Froude number, however, if a ship moves in shallow water with different depth, the flow may exist several depth Froude numbers. For a dredged channel, which is depicted in Fig. 1, its flow exists two depth Froude numbers, if the low depth Froude number is subcritical and the high one is supercritical, the flow can be called as the sub-supercritical mixed flow; if the low depth Froude number is supercritical and the high one is also supercritical, the flow can be called as the super-supercritical mixed flow, and so on.

The researches on shallow-water wave problem have practical significance in shipbuilding, ocean, coastal and hydraulic engineering. The mutual effects among ship wave, sidewall and seabed in restricted waterways may cause the sinkage and trim of ship, and affect ship resistance and its safe navigation, which may also lead oceanic and coastal structure to damage (Tuck, 1967; Chen and Sharma, 1995; Gourlay, 2000). Based on Michell shallow-water wave equation, the hydrodynamic forces of a slender ship moving at subcritical or supercritical speed in open water were studied (Tuck, 1966; Gourlay, 2008). Then, without considering the dispersive and nonlinear effects, Tuck (1967) further extended his researches to the finite-width channel, he calculated the ship sinkage of various width, and indicated that the effect of finite width was far more serious for sinkage than it was for trim. Gourlay (2008) and Zhang et al. (2015) further extended the methods of Tuck (1966) and Beck et al. (1975) to rectangular canal, dredged channel and stepped canal; Gourlay (2008) solved the sub-subcritical ship hydrodynamic problems, and Zhang et al. (2015) established the analytical models of SHPF at sub-subcritical, sub-supercritical and super-supercritical speeds. However, the above calculated results may be reasonable for the depth Froude number far from 1, because of the ignorance of dispersive and nonlinear effects in Michell equation. With considering the dispersive effect, the hydrodynamic problems caused by a slender ship moving at subcritical or supercritical speed in open water were solved (Mei, 1976; Gourlay and Tuck, 2001). Furthermore, the KP and Boussinesq equations with dispersive, nonlinear and unsteady effects had been used to calculate the solitary wave and hydrodynamic force caused by ship moving at transcritical speed in rectangular canal (Chen and Sharma, 1995, 1997; Jiang, 2001).

Variation of pressure caused by a moving ship is generally called as ship hydrodynamic pressure field (SHPF), and the pressure characteristics can be used for military purposes of discovering or identifying ship. At present, most researches on SHPF mainly aim at the constant depth or open water, and rarely concern the non-uniform depth, mixed flow and so on (Sahin and Hyman, 2001; Zhang et al., 2002; Lazauskas,

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http://dx.doi.org/10.1016/j.oceaneng.2017.01.028
Received 4 August 2016; Received in revised form 27 January 2017; Accepted 29 January 2017
Available online 07 February 2017

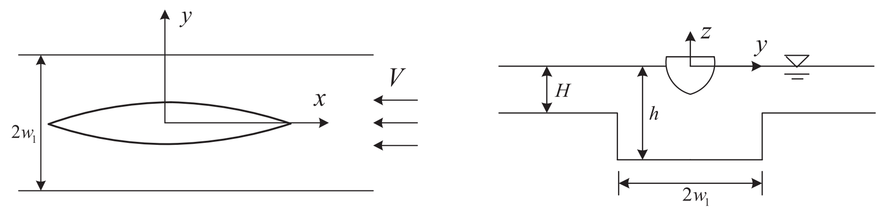

Fig. 1. Coordinate system of dredged channel.

2007; Deng et al., 2014a, 2014b). Based on the above-mentioned research works (Mei, 1976; Beck et al., 1975; Gourlay, 2008; Zhang et al., 2015), we derive the new analytical models of sub-supercritical SHPF with the dispersive effect in dredged channel , and extend the pressure calculation of ship hull surface to that of the whole flow field, the sub-subcritical mixed flow to the sub-supercritical one in inner and outer regions. The distribution characteristics of sub-supercritical SHPF in dredged channel can be obtained by calculation and analysis, and the calculated results have been compared with the typical experimental ones. Meanwhile, the SHPF analytical models with the dispersive effect in open water, rectangular canal or stepped canal can also be derived by further simplifying or adopting similar method.

2. Governing equations

As is depicted in Fig. 1, we consider that a ship moves along the centerline in a dredged channel, with supposing the ship speed is V, its length is L (or $2l$), and its width is $2b$. The dredged channel can be divided into the inner and outer regions, the depth of inner region is h, its depth Froude number is F_h, its width is $2w_1$, and the depth of outer region is H, its depth Froude number is F_H, because of $h > H$, then $F_h < F_H$. For $F_h = V/\sqrt{gh} < 1$ and $F_H = V/\sqrt{gH} > 1$, it can be called as the sub-supercritical mixed flow in dredged channel. A Cartesian coordinate system $oxyz$ is employed and moves with ship, the origin o locates at the center of hull waterline, with the axis z vertically upward, and the axis x in the direction of ship motion. We consider only the region $y \geq 0$, because of the flow symmetry about $y = 0$.

Supposing the fluid is inviscid, incompressible and irrotational, meanwhile, ignoring the unsteady effect, a steady wave equation can be derived (Jiang, 2001; Zhang and Gu, 2006), i.e.,

$$(1 - F_h^2)\varphi_{xx} + \varphi_{yy} + \frac{3V}{gh}\varphi_x\varphi_{xx} + \frac{F_h^2 h^2}{3}\varphi_{xxxx} = 0 \tag{1}$$

where φ is the local-depth-averaged perturbation velocity potential, g is the gravitational acceleration, the third and the fourth terms represent the nonlinear and the dispersive effects respectively.

If the ship is slender, the nonlinear effect can generally be ignored, and the dispersive effect should be considered, which can present the wave patterns of high-speed ship. Here $\phi(x, y)$ is known as the local-depth-averaged perturbation velocity potential of inner region, and $\Phi(x, y)$ is that of outer one. Thus, the governing equations in inner and outer regions with the dispersive effect can be expressed as follows respectively,

$$\beta_1^2\phi_{xx} + \phi_{yy} + \gamma_1^2\phi_{xxxx} = 0 \text{ as } F_h < 1 \tag{2}$$

where $\beta_1 = \sqrt{1 - F_h^2}$, $\gamma_1 = F_h h/\sqrt{3}$.

$$\beta_2^2\Phi_{xx} - \Phi_{yy} - \gamma_2^2\Phi_{xxxx} = 0 \text{ as } F_H > 1 \tag{3}$$

where $\beta_2 = \sqrt{F_H^2 - 1}$, $\gamma_2 = F_H H/\sqrt{3}$.

The hull boundary condition can be written as,

$$\phi_y(x, 0) = -Vf_x(x) \text{ as } |x| \leq l \tag{4}$$

where $y = f(x)$ is the equation of a slender ship hull.

The flow on the interface of inner and outer regions should satisfy the continuous matched conditions that the local-depth-averaged perturbation velocity potential and transverse volume flux are equal respectively (Gourlay, 2008; Zhang et al., 2015),

$$\phi_x(x, w_1 - 0) = \Phi_x(x, w_1 + 0) \text{ and } h\phi_y(x, w_1 - 0) = H\Phi_y(x, w_1 + 0) \tag{5}$$

Meanwhile, for the inner subcritical flow, the upstream and downstream boundaries at infinity should satisfy the condition of perturbation attenuation. And for the outer supercritical flow, it is necessary to set the moving backwards condition of ship wave.

Solving the above mathematical problems by using the Fourier integral transform method, and the transformation and inverse transformation forms of Fourier integral in inner and outer regions can be expressed as follows respectively,

$$\tilde{\phi}(k, y) = \int_{-\infty}^{\infty} \phi(x, y)e^{ikx}\mathrm{d}x,\ \tilde{\Phi}(k, y) = \int_{-\infty}^{\infty} \Phi(x, y)e^{ikx}\mathrm{d}x \tag{6}$$

$$\phi(x, y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{\phi}(k, y)e^{-ikx}\mathrm{d}k,\ \Phi(x, y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{\Phi}(k, y)e^{-ikx}\mathrm{d}k \tag{7}$$

The velocity and pressure field can be calculated after obtaining the local-depth-averaged perturbation velocity potential. If ignoring the third term of Eqs. (2) and (3), the SHPF analytical models without the dispersive effect in restrict waterways can be derived, which have been given in the authors' previous article (Zhang et al., 2015).

Noting $\sigma_1 = (\gamma_1^2k^2 - \beta_1^2)k^2$,$\sigma_2 = (\gamma_2^2k^2 + \beta_2^2)k^2$, then

$$t_1 = k\sqrt{\beta_1^2 - \gamma_1^2k^2} \text{ as } 0 < k < \beta_1/\gamma_1 \tag{8}$$

$$s_1 = k\sqrt{\gamma_1^2k^2 - \beta_1^2} \text{ as } k > \beta_1/\gamma_1 \tag{9}$$

$$t_2 = k\sqrt{\gamma_2^2k^2 + \beta_2^2},\ d_1 = Ht_2/(ht_1),\ q_1 = Ht_2/(hs_1) \tag{10}$$

Using the Fourier integral transform and the condition of perturbation attenuation, Eq. (2) of inner region can become,

$$\tilde{\phi}_{yy}(k, y) + \sigma_1\tilde{\phi}(k, y) = 0 \text{ as } y < w_1 \tag{11}$$

When k changes from $-\infty$ to $+\infty$, the solutions of Eq. (11) have two cases: $\sigma_1 > 0$ or $\sigma_1 < 0$, which need to be discussed respectively.

(1) For $\sigma_1 > 0$, there is $\gamma_1^2k^2 - \beta_1^2 > 0$, i.e. $k > \beta_1/\gamma_1$ or $k < -\beta_1/\gamma_1$, and the general solution of Eq. (11) can be written as,

$$\tilde{\phi}(k, y) = A(k)e^{iy\sqrt{\sigma_1}} + B(k)e^{-iy\sqrt{\sigma_1}} \tag{12}$$

where $A(k)$ and $B(k)$ are undetermined coefficient.

Using Eqs. (6), and (4) can become,

$$\tilde{\phi}_y(k, 0) = iVk\tilde{f}(k) \tag{13}$$

where $\tilde{f}(k) = \int_{-\infty}^{\infty} f(x)e^{ikx}\mathrm{d}x$.

Taking $f(x) = \begin{cases} b[1 - (x/l)^2], & |x| \leq l \\ 0, & |x| \geq l \end{cases}$, then $\tilde{f}(k) = -\frac{4b}{l^2}\left[\frac{l\cos(kl)}{k^2} - \frac{\sin(kl)}{k^3}\right]$, which is the even function about k, and where $b = S_{max}/(2h)$ for shallow water , S_{max} is the maximum cross-sectional area of ship.

Combing Eqs. (12) with (13), we can get,

$$\tilde{\phi}(k, y) = 2B(k)\cos(y\sqrt{\sigma_1}) + \frac{kV\tilde{f}(k)e^{iy\sqrt{\sigma_1}}}{\sqrt{\sigma_1}} \tag{14}$$

(2) For $\sigma_1 < 0$, there is $\gamma_1^2k^2 - \beta_1^2 < 0$, i.e. $-\beta_1/\gamma_1 < k < \beta_1/\gamma_1$, and the