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Optimal sensor configuration for positioning seafloor geodetic node

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ABSTRACT

This paper deals with the optimal acoustic sensor configuration for positioning seafloor geodetic node. We use the determinant of Fisher Information Matrix (FIM) to measure the performance of different sensor configuration. This criteria is calculated in both 2D and 3D scenarios on a more practical assumption that range observations have different weights depending on their value, instead of identical weights. Then the uncertainty of initial node position is also taken into consideration for the calculation of determinant of FIM which was often neglected in the optimizing process. We present a kind of optimal configurations based on the maximum of *determinant* (FIM), which is circular and has different optimal radius at each corresponding scenario. Simulative experiments are carried out to demonstrate the optimal configuration. Practical experiments in South China Sea can further prove the relationship between positioning accuracy, configuration and determinant of FIM.

1. Introduction

Last two decades have witnessed a rapid development in seafloor observatory science. Independent seafloor networks are built and maintained by many country in the world, such as Regional Scale Nodes (RSN) by USA, Dense Oceanfloor Network system for Earthquakes and Tsunamis (DONET) by Japan, North East Pacific Time-series Seafloor Networked Experiments (NEPTUNE) and Victoria Experimental Network Under the Sea (VENUS) by Canada, and European Seas Observatory Network (ESONET) by EU (Favali et al., 2010), as an important mean to observe and monitor the ocean environment.

Seafloor geodetic node is a fundamental component of these networks, and acts as a reference for seafloor motion and scanning. Therefore, a main effort is to determine the node position, by integrated methods using Global Navigation Satellite System (GNSS) and acoustic sensors, such as Ultra Short Baseline (USBL), Short Baseline (SBL) and Long Baseline (LBL) (Leonard and Bahr, 2016; Seto et al., 2013).

In this paper, we look for a group of optimal acoustic sensor configurations to ensure the highest positioning accuracy of seafloor geodetic node. There are two popular indexes in navigation and positioning system to measure the configuration, Geometric Dilution of Precision (GDOP) and the determinant of FIM (Martínez and Bullo, 2006). GDOP is square root of the trace of Cramer Rao Lower Bound (CRLB), which is the inverse of FIM (Sharp et al., 2009). Moreover, the two indexes are equivalence to some extent that the minimum GDOP and the maximum *determinant* (FIM) both represent the highest positioning accuracy. The determinant of FIM is selected in this paper because it is easier to calculate without inversing matrix.

Nowadays, there are many works on optimizing sensor configuration. Zhang (1992) discussed the relationship between sensor configuration and positioning accuracy in 2D scenario, and gave several conditions with fewer than three sensors. In his later work, the conditions were developed as to satisfy multiple-sensor-configuration (Zhang, 1995). Bishop et al. (2007) gave the conditions using Time of Arrival (TOA) observations in 2D scenario, and showed several detailed configurations composed of three and four sensors respectively. Zhao et al. (2013) expanded this problem to both 2D and 3D scenarios using both bearing and ranging observations, and gave examples for 2D and 3D scenarios using regular polygons and Platonic solids respectively. Methods are also developed for solving dynamic configurations based on genetic algorithm or particle filter, but they more focused on dynamic optimizing algorithm than detailed configuration (Majid and Joelianto, 2012; Ding et al., 2014).

There are infinite solutions for this optimizing problem when

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sensors are more than four, considering rotation and combination of configurations (Xue and Yang, 2017). Researchers tended to give limiting conditions for the problem, instead of providing detailed examples of optimal configurations, unless sensors are equal to or less than four. Moreover, most of the existing works considered that the observations have identical weights, which is not practical in seafloor positioning (Martínez and Bullo, 2006). Furthermore, the optimal configuration is obtained based on an initial node position with uncertainty, so the Probability Density Function (PDF) of the seafloor node should be taken into consideration together. We want to set different weights for observations and make use of the initial PDF in this paper, as to present detailed optimal configuration in more general scenario.

Our work are also inspired by researches about optimal GNSS satellites configuration. GDOP and determinant of FIM were used to select satellites for fast positioning (Sharp et al., 2009; Wang, 2005). Optimizing problem was also discussed for multi-GNSS constellations (Han et al., 2013; Teng and Wang, 2014; Teng et al., 2015; Xue et al., 2015). Particularly, a systematical contribution was made by Xue et al. (2015) to solve the optimizing problem analytically, and three kinds of basic configurations were presented in 3D scenario, cone configuration, Descartes configuration, and Walker configuration. Three kinds of solutions for minimizing GDOP depend on the number of sensors, infinite solutions, finite solutions and no solution (Xue and Yang, 2017). But there are some features should be noticed in this project other than the characteristics of GNSS application: a) unknown parameters do not include receiver clock offset; b) the measurement error is assumed as a white Gaussian noise with variance dependent on range; c) acoustic sensors are placed on a horizontal plane, and approximately $z = 0$.

The remainder of this paper is organized as follows. Section 2 analyses the positioning method for seafloor geodetic node, and calculates the value of *determinant* (FIM) and GDOP. Section 3 provides optimal sensor configuration for 2D and 3D scenarios by maximizing *determinant* (FIM), and the optimal sensor configuration by introducing the PDF of seafloor node. Some simulative experiments and practical experiments are carried out in Section 4. Finally, conclusions are drawn in Section 5.

2. Integrated positioning method using GNSS and acoustic sensors

2.1. Analysis of underwater positioning sensors

GNSS and acoustic sensors are installed in surface buoys or surveying ships. If the coordinates of GNSS and acoustic sensors in vehicle coordinate system are measured, and the attitudes of the buoys or ships are provided by Gyro, the positions of acoustic sensors can be obtained by some computations based on the GNSS coordinates.

The most common acoustic sensors are USBL, SBL and LBL, distinguished according to their baseline, as shown in Table 1(Leonard and Bahr, 2016; Tan et al., 2011).

In the case of positioning seafloor geodetic node, LBL sensor is chosen to be installed in the node for its good performance in deep sea environment. The position of underwater LBL sensor is calculated as shown in Fig. 1. Range measurement is calculated by the product of mean sound speed and Time of Arrival (TOA). Many LBL sensors provide time delay error for the calculation of accurate range, therefore it is not necessary to get Time Difference of Arrival (TDOA) by subtracting two TOAs.

Table 1
Acoustic Sensors.

Acoustic sensor	Length of Baseline	Measurement
USBL	< 10 cm	Range and Bearing
SBL	20–50 m	Range
LBL	50–6000 m	Range

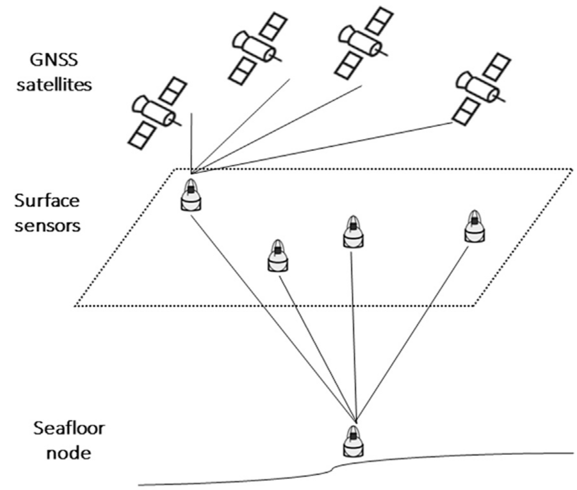

Fig. 1. Positioning seafloor geodetic node using GNSS and acoustic sensors.

2.2. Determinant of FIM and GDOP

Suppose that n sensors are put on sea surface as surface nodes. For the convenience of discussion, we assume that sea surface is quiet, and let $p_i = (x_{pi}, y_{pi}, 0)$ denotes the position of i-th surface node. Let $q = (x_q, y_q, z_q)$ denotes the position of seafloor node. Let r_i denotes the observed range between i-th surface node and seafloor node, and $\hat{r}_i$ denotes the true value of this range.

The functional model and random model for the positioning process are expressed respectively as

$$r_i = \|p_i - q\|_2 + w_i, \quad E(w_i) = 0 \tag{1a}$$

$$\hat{r}_i \sim N(r_i, \sigma_i^2), \quad \sigma_i = a + br_i \tag{1b}$$

where a and b are the prior information to evaluate the relationship between r_i and σ_i, and can be provided by the performance of LBL sensor in certain underwater environment. Assume that range r_i is not correlated with another, and variance σ_i^2 is dependent on r_i.

Eq. (1) can be linearized as

$$r_i - \|p_i - q\|_2 = e_i \delta q + w_i \tag{2}$$

where δq is the difference between initial position and estimated position of seafloor node, and e_i is the line-of-sight vector.

$$e_i = \frac{\partial \|p_i - q\|_2}{\partial q} = (x_{pi} - x_q \quad y_{pi} - y_q \quad z_{pi} - z_q)/\|p_i - q\|_2 \tag{3}$$

The estimated q can be obtained by following iterations until δq reaches a lower bound.

$$\delta q = \arg\min(V^T P V) \tag{4a}$$

$$q = q + \delta q \tag{4b}$$

where $V = (w_1 \quad w_2 \quad \cdots \quad w_n)^T$ and $P = diag(1/(a + br_1)^2 \quad 1/(a + br_2)^2 \quad \cdots \quad 1/(a + br_n)^2)^T$.

Solving the equation system (2) based on the least square criterion, we can get the expression of FIM and GOOP (Chaffee and Abel, 1994).

$$F = \sum_{i=1}^{n} e_i e_i^T / \sigma_i^2 \tag{5a}$$

$$\mathrm{GDOP} = \sqrt{tr(F^{-1})} = \sqrt{tr\left[\left(\sum_{i=1}^{n} e_i e_i^T / \sigma_i^2 \right)^{-1} \right]} \tag{5b}$$