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Applicability and limitations of highly non-linear potential flow solvers in the context of water waves

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ABSTRACT

This paper presents a general overview of the practical applicability of non-linear potential flow solvers for water wave propagation. Those numerical models are unable to describe explicitly the wave breaking phenomena, including free surface reconnexion, energy dissipation, etc. This reduces their range of application in real sea waves, which are in the most general context irregular and directional wave fields. This study covers the influence of discretization, water depth, directional spreading and spectral peakedness on the limitations in the use of such non-linear potential models. The approach at use relies on a highly non-linear model based on the High-Order Spectral method.

1. Introduction

The fine description of the water waves propagation is necessary to ocean engineers, especially in the context of the numerical simulation of wave-structure interactions. More and more efforts are dedicated to the accurate description of extreme waves (and their possible interactions with structures at sea). In this concern, highly and fully non-linear potential flow solvers are the dedicated models for the description of the wave environment. For wave-structure interactions, the usual approach, once the surrounding wave-field is accurately described, is to couple such non-linear wave model with a specific model dedicated to the solution of the interactions. Depending on the main physical processes at play, these can use e.g. non-linear potential flow, Reynolds-Average Navier-Stokes (RANS) equations, Smooth Particle Hydrodynamics (SPH), etc. Several ways to perform the coupling exist: spatial, temporal or mathematical decomposition (the so-called SWENSE approach, see e.g. Luquet et al., 2007). The use of such procedures is still necessary for the practical solution of wave-structure interactions, primarily due to the computational effort inherent to the direct solution of the problem thanks to advanced Computational Fluid Dynamics (CFD) solvers.

At the same time, the study of non-linear wave processes is also of interest with an increased concern with the extreme sea-states as well as with the formation of large waves in a given sea state (the so-called freak or rogue waves Haver, 2004). The accurate description of the corresponding wave fields is accessible through the non-linear potential flow solvers. Consequently, they are now popular to answer the previous needs and for practical applications in the context of wave-structure interactions.

However, the formalism adopted is limited to non-breaking waves, reducing the range of application in real sea waves. This limitation is mainly due to: i) the irrotational flow assumption which is violated when having free surface reconnection, air entrapment, etc. (see e.g. Lubin et al., 2006) and ii) the inviscid fluid assumption, which does not allow to take care of the consequent dissipation of energy. We indicate that some efforts are currently done to model the main effects of the breaking process on the wave field evolution (see e.g. Perlin et al., 2013). However, we restrict our study to original nonlinear potential flow solvers without such modeling.

This papers intends to present the practical applicability of such models with respect to the different wave conditions. These are usually described as irregular directional wave fields. The possible limitation of the solvers will be assessed with the equivalent of scatter diagrams, i.e. validity with respect to the significant wave height H_s and peak period T_p of the corresponding sea state. The proposed study covers the influence of numerical discretization, water depth, angular and frequency spreading.

The approach used here relies on highly non-linear numerical simulations based on the High-Order Spectral (HOS) method initiated by West et al. (1987) and Dommermuth and Yue (1987). This method has been widely validated and applied to several configurations, e.g. modulational instabilities Fernandez et al. (2014); Toffoli et al. (2010a), nonlinear energy transfers Tanaka (2001), bi-modal seas Toffoli et al. (2010b), freak waves Ducrozet et al. (2007); Xiao et al. (2013); Sergeeva and Slunyaev (2013) among others. This method can consequently be considered as a good example of non-linear potential flow solvers: it is mature and accessible to practical engineering

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applications. The implementation provided in the open-source HOS-ocean code Ducrozet et al. (2016) will be used throughout the study.

The paper will be divided as follows, the first part presents the non-linear wave propagation model based on the HOS method and the different initial conditions accessible within the model. Then, the second part focuses on the methodology chosen to assess the limitations of the non-linear simulations as well as some existing theoretical and experimental results concerning the presence of breaking events in the different configurations. In the end, the last part focuses on the description of the influence of some of the key parameters for regular and irregular sea states applicability ranges. This part will enable an overview of the practical capabilities of the non-linear potential flow solvers.

2. Numerical wave model

The study of large-scale and long-term evolution of non-linear directional wave fields relies on the existence of accurate and efficient numerical methods. In this concern, highly and fully non-linear potential flow solvers are the dedicated models. Indeed, in non-breaking wave conditions, the essential physics are included in the non-linear potential formalism. Within this class of approximation, several different numerical approaches exist that are briefly reminded hereafter. We also indicate that in such models, different methodologies can be used to take into account the effect of the wave breaking on the sea-state evolution. However, this is still an active field of research (e.g. Perlin et al., 2013) and a definite model is still needed. In this study, we consequently restrict the field to nonlinear potential flow solvers without any dissipation model.

The whole numerical domain may be discretized (i.e. in horizontal and vertical directions) leading to volume methods. Usually, after the application of a σ-transform, such models solve efficiently the problem in a fixed computational domain. We refer for instance to the development of OceanWave3D model based on a finite-difference discretization of the volume problem Bingham and Zhang (2007); Engsig-Karup et al. (2009).

An alternative in potential flow formalism is to solve the problem on its boundaries, reducing the number of unknowns. Those methods are referred as Boundary Element Methods (BEM). Up to now, the main limitation with these is the computational effort which appears as prohibitive for large-scale and long-term computations (even with the latest developments of Fast Multipole Algorithms, see e.g. Fochesato and Dias, 2006).

In the end, the problem may be solved on the free surface only usually making use of the Hamiltonian formulation of the free surface problem. Efficient and accurate solution of this surface problem is usually achieved thanks to pseudo-spectral methods, namely High-Order Spectral (HOS) or Dirichlet to Neumann Operator (DNO) approaches (which are equivalent, see Schäffer, 2008). The main limitation is here associated to the geometry that has to be simple (usually rectangular domain in horizontal directions and constant water depth).

In this section, the highly non-linear potential flow wave model at use to assess the range of applicability of this class of method is described. The detailed set-up of the numerical computations using this HOS method is also given with a specific attention paid to the initial conditions.

2.1. High-Order Spectral method

In this study, all numerical simulations are performed with the HOS method. This method has been initiated by West et al. (1987) and Dommermuth and Yue (1987). Open-source HOS-ocean code Ducrozet et al. (2016) will be used and we refer to the different publications related to this model for examples of validation Bonnefoy et al. (2009); Ducrozet et al. (2007).

We consider a rectangular fluid domain D of horizontal dimensions $L_x \times L_y$ and constant water depth h associated with a Cartesian coordinate system. Its origin O is located at one corner of the domain with (Oxy) representing the horizontal axes and (Oz) the vertical one oriented upward with z=0 located at the mean free surface. We are working under the potential flow theory (assuming the fluid to be incompressible and inviscid and the flow irrotational). Then, continuity equation reduces to the Laplace equation for the velocity potential ϕ.

Boundary conditions close the system of equations. Periodicity is assumed in horizontal directions x and y. These lateral boundary conditions, associated to the bottom one, allow us to define a spectral basis on which the velocity potential in the whole volume will be expanded. At the same time, surface quantities are also expressed on a spectral basis allowing the use of Fast Fourier Transforms (FFTs). We consider the free surface boundary conditions which are written, following Zakharov (1968), using surface quantities namely the free surface elevation η and the free surface velocity potential $\tilde{\phi}(x, y, t) = \phi(x, y, \eta(x, y, t), t)$. $z = \eta(x, y, t)$ describes the free surface position, assuming no wave breaking occurs (i.e. the free surface is a single valued function of x and y). The kinematic and dynamic free surface boundary conditions then read

$$\frac{\partial \eta}{\partial t} = (1 + |\nabla\eta|^2)W - \nabla\tilde{\phi}\cdot\nabla\eta \tag{1}$$

$$\frac{\partial \tilde{\phi}}{\partial t} = -g\eta - \frac{1}{2}|\nabla\tilde{\phi}|^2 + \frac{1}{2}(1 + |\nabla\eta|^2)W^2 \tag{2}$$

with $\nabla$the horizontal gradient and $W(x, y, t) = \frac{\partial \phi}{\partial z}(x, y, \eta, t)$ the vertical velocity at the free surface. This is the only quantity beyond the system (1)–(2) which needs solution in the water bulk. Later on W will be evaluated thanks to the order consistent HOS scheme of West et al. (1987). Once the vertical velocity is evaluated, it is possible to advance in time the two unknowns η and $\tilde{\phi}$ thanks to an efficient 4$^{\text{th}}$ order Runge-Kutta Cash-Karp scheme with adaptive step size Cash and Karp (1990).

The HOS procedure relies on a series expansion in wave steepness ϵ up to the so-called HOS order M of the velocity potential. Expanding a Taylor series around z=0 and collecting terms at each order in wave steepness leads to a triangular system. A similar series expansion for the vertical velocity leads to another triangular system, which is solved iteratively. This enables an arbitrary choice of the order of nonlinearity M.

The resulting numerical method is pseudo-spectral and exhibits very interesting convergence properties: exponential convergence rate with respect to the horizontal discretization as well as with the HOS order M Bonnefoy et al. (2009). Thus, this HOS model features high efficiency and accuracy compared to other advanced methods for wave propagation, see Ducrozet et al. (2012).

2.2. Initial conditions

The HOS method allows the accurate and efficient propagation of wave fields with possibly high level of non-linearity. As stated previously, the method has been widely validated in several configurations, e.g. nonlinear energy transfers Tanaka (2001), modulational instabilities Fernandez et al. (2014); Toffoli et al. (2010a), bi-modal seas Toffoli et al. (2010b) or freak waves Ducrozet et al. (2007); Xiao et al. (2013); Sergeeva and Slunyaev (2013).

Then, the numerical simulations rely on the definition of relevant initial conditions of interest. Once the free surface elevation $\eta(x, y, t = 0)$ and velocity potential $\tilde{\phi}(x, y, t = 0)$ are specified, the two unknowns are advanced in time with the previous HOS procedure. Within HOS-ocean, different kind of initial conditions are accessible, starting from regular waves up to directional irregular sea states.

The regular waves are initialized thanks to the nonlinear regular wave solution of Rienecker and Fenton (1981), based on the stream