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Free flexural vibration of a cylindrical shell horizontally immersed in shallow water using the wave propagation approach

Peng Wang, Tianyun Li, Xiang Zhu*

School of Naval Architecture and Ocean Engineering, Huazhong University of Science and Technology, Wuhan 430074, China

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ABSTRACT

The free flexural vibration of a cylindrical shell horizontally immersed in shallow water is analyzed in low frequency range using the wave propagation approach. The effects of both the upper and lower fluid boundaries, *i.e.*, free surface and seabed, are considered with the image method, and the scattering effect of cylindrical shell is neglected. Motions of cylindrical shell and fluid are modeled with the Flügge shell theory and wave equation, respectively. The accuracy of present method is verified through comparison with available experimental works. The modal added mass is introduced to show the influence of shallow water on coupled modal frequency. The influence of shallow water is significant in the case of small water depth and it's the contributions from fluid boundaries. The presence of free surface (seabed) will make negative (positive) contribution to the modal added mass and finally result in the increase (reduction) of coupled modal frequency. But the individual fluid boundary effect will be reduced gradually and finally negligible when the distance between cylindrical shell and fluid boundary increases. The influence of shallow water can not be negligible when the water depth is less than 8 times of shell radius. This work will help to select proper test environment for submerged cylindrical shell.

1. Introduction

Cylindrical shells have wide applications in ocean engineering areas such as submarines, pipelines and vehicles, and they are usually submerged in bounded flow field, *e.g.*, the shallow water. When a submarine is sailing in the sea, especially in the epicontinental sea, the multiple reflections of acoustic wave by both the upper and lower fluid boundaries will make the vibro-acoustic characteristics of immersed shell quite different from those of the shell ideally immersed in infinite flow field. To ensure good dynamic performance of underwater structures, *i.e.*, submarine pressure hulls and the offshore pipelines, a better understanding of the vibration characteristics of cylindrical shell immersed in a bounded flow field, *e.g.*, shallow water, is in some cases crucial.

When bounded by only one plane boundary, the flow field would be a half-space or semi-infinite field. Concentrated on the dynamic characteristics of shells immersed in half-space bounded by the free or rigid surface, various studies have been carried out using different methods, such as the bipolar transformation technique (Chang and DiMaggio, 1971), the combination of Helmholtz integral approach and Fourier transform technique (Skidan et al., 1974), the boundary integral method (Seybert and Soenarko, 1988; Seybert et al., 1985; Rizzo et al., 1985; Liu and Rizzo, 1993), the boundary element method (Li et al., 1994), the wave domain approach (Ye et al., 2013; Li et al., 2014). Available works on flexural vibration of cylindrical shell partially coupled with fluid mainly present in two topics. One is the free vibration of finite cylindrical shell partially coupled with fluid, the Rayleigh-Ritz method (Amabili and Dalpiaz, 1995; Amabili, 1996, 1997; Ergin, 1997) and boundary integral method (Ergin and Temarel, 2002) are proposed. Zhang (2014) extended his research on the sound radiation of finite cylindrical shell semi-submerged in fluid with the wave domain approach (WDA). The other is the sound radiation of infinite cylindrical shell partially coupled with fluid, the Fourier series expansion technique (Salaün, 1991; Wang et al., 2015) and the WDA (Li et al., 2003) are applied to solve this problem.

When the fluid is bounded by multiple plane boundaries, the flow field could be finite, *e.g.*, the shallow water is bounded by the free surface and seabed. The motions of a ship sailing in shallow water have been widely studied, *e.g.*, Tuck (1970), Oortmerssen (1976), Andersen (1979) and Inglis and Price (1980). Oğuz (1996) studied the sound emission by a finite object in shallow water. With the application of three dimensional hydroelastic approach, Ergin et al. (1992) investigated the influence of free or rigid surface on the dynamic characteristics of submerged cylinder, and an experiment was also conducted to verify the accuracy of theoretical approach. To the best of author's knowledge, few analytical work has been done about the free flexural

* Corresponding author.
E-mail address: zhuxiang@hust.edu.cn (X. Zhu).

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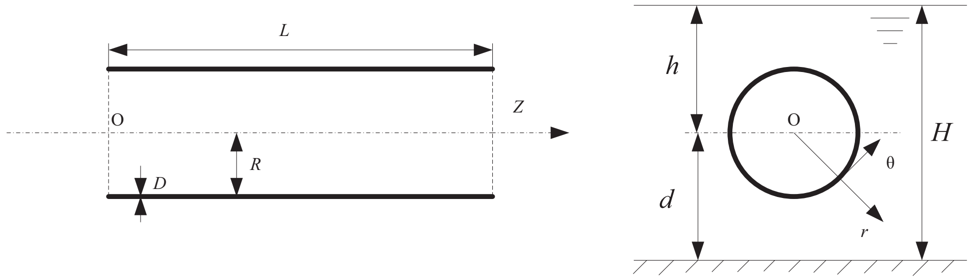

Fig. 1. A cylindrical shell immersed in shallow water and the coordinate system.

vibration of finite cylindrical shell submerged in shallow water, except the work done by Ergin et al. (1992). Here, the authors attempt to put forward an analytical method for solving this problem in low frequency range.

When considering a cylindrical shell vibrates in the fluid field, the fluid coupled with shell is generally compressible, and it's usually assumed to be an acoustic media. In this case, the reaction of fluid on structures could be presented by the sound pressure at the interface. Therefore, the vibration of fluid-structure coupled system could be considered as a sound-structure interaction problem. Zhang et al. (2001a, 2001b) and Zhang (2001, 2002a, 2002b) have successfully applied the wave propagation approach (WPA) to study the free flexural vibration of cylindrical shell *in vacuo*, filled with or submerged in fluid. And the WPA has been proved to be a simple, effective and non-iterative method to solve the sound-structure interaction problem with good accuracy. According to the WPA, the dynamic solutions for the shell could be expressed in a wave propagation form. Besides, the image method has been proved to be effective to deal with the problem of structures vibrating in bounded flow field, *e.g.*, Gaunaurd and McCarthy (1987), Gaunaurd and Huang (1996) and Huang and Gaunaurd (1997). Here, taking the advantages of both the WPA and image method, the authors aim at proposing an analytical solution for the free flexural vibration of a finite thin cylindrical shell immersed in shallow water in low frequency range.

The fluid-structure coupled system may have different characteristics in different frequency ranges, and the application of appropriate shell theory is of great importance. There are a variety of shell theories established to describe the motion of thin cylindrical shells (Leissa, 1993), and the Flügge shell theory (Flügge, 1973) is one of the typical theories with good accuracy in low frequency range. However, shear deformations and the rotations may affect the vibration characteristics of cylindrical shells in high frequency range. In that case, proper shell theory should be employed in order to obtain the accurate vibration characteristics. Magniez et al. (2014) adopted the first shear deformation shell theory to study the sound transmission through mixed 3D sandwich cylinders in high frequency range. Daneshjou et al. (2009) applied the first-order shear deformation theory to analyze the sound transmission loss of orthotropic cylinder with the subsonic external flow. Then, Daneshjou et al. (2016) predicted the sound transmission loss of thick shell in a wide frequency range with the three dimensional theory of elasticity. Daneshjou et al. (2016) also pointed out that the effects of shear deformation and rotary inertia are mainly significant in high frequency range. Zhou et al. (2014) used the Love thin shell theory to investigate the sound transmission through a thin cylindrical shell subjected to external mean flow. Here, this work mainly focuses on the free flexural vibration of cylindrical shell horizontally immersed in shallow water in low frequency range. So the Flügge thin shell theory is adopted, and the effects of shear deformation and rotary inertia are neglected.

In this work, an analytical method is proposed for the free flexural vibration of a cylindrical shell horizontally immersed in shallow water in low frequency range. The effects of both the upper and lower fluid boundaries, *i.e.*, the free surface and seabed, are considered to model the acoustic radiation problem in the absence of external incident waves, so the scattering effect of cylindrical shell is omitted here to simplify the problem just as works done by Seybert and Soenarko (1988), Seybert et al. (1985), Rizzo et al. (1985), Liu and Rizzo (1993), Li et al. (2014) and Ye et al. (2013). The motion of cylindrical shell is modeled with the Flügge thin shell theory and wave propagation approach, and that for the fluid is modeled with the wave equation and image method. The modal added mass is also introduced for problem here to reveal the influence mechanism of shallow water on coupled modal frequency. This work not only gives a better understanding of the special vibration characteristics of immersed cylindrical shell in shallow water, but also helps to select proper test environment for submerged cylindrical structures.

2. Theoretical analysis

The specific problem discussed here is a thin cylindrical shell vibrating in shallow water with its axis parallel to both the upper and lower fluid boundaries. It's assumed to be vacuum inside the shell. Fluid surrounding the shell is assumed to be an acoustic media, and the flow field is bounded by both the upper and lower fluid boundaries, *i.e.*, the upper one is generally the free surface and the lower one may be the seabed or ocean bottom. The sketch of a cylindrical shell horizontally immersed in shallow water is shown in Fig. 1. The shell is of length L, mid radius R, and thickness D. The material of shell is of density ρ_s and elastic modulus E. The density of fluid is ρ_f. The water depth of shallow water is H, the axis of cylindrical shell is of a distance h away from the upper fluid boundary and a distance d away from the lower fluid boundary. Obviously, the geometric relation is satisfied, $H = h + d$. h is also called the immersion depth of cylindrical shell. A cylindrical coordinate is selected, and z, r, θ are the axial, radial and circumferential directions, respectively.

2.1. Motion of shell

Since this work mainly focuses on the flexural vibration of a thin cylindrical shell submerged in shallow water in low frequency range, the effects of both shear deformation and rotary inertia are neglected. Therefore, the stress equilibrium equation of cylindrical shell immersed in shallow water could be expressed with the Flügge shell theory (Flügge, 1973)

$$\begin{cases} \dfrac{\partial N_z}{\partial z} + \dfrac{\partial N_{\theta z}}{R\partial\theta} = \rho_s D\dfrac{\partial^2 u}{\partial t^2} \\ \dfrac{\partial N_{\theta z}}{\partial z} + \dfrac{\partial N_\theta}{R\partial\theta} - \left(\dfrac{\partial M_{z\theta}}{R\partial z} + \dfrac{\partial M_\theta}{R^2\partial\theta}\right) = \rho_s D\dfrac{\partial^2 v}{\partial t^2} \\ \dfrac{\partial^2 M_z}{\partial z^2} + \dfrac{\partial^2 M_{z\theta}}{R\partial z\partial\theta} + \dfrac{\partial^2 M_{\theta z}}{R\partial z\partial\theta} + \dfrac{\partial^2 M_\theta}{R^2\partial\theta^2} + \dfrac{N_\theta}{R} - p|_{r=R} = \rho_s D\dfrac{\partial^2 w}{\partial t^2} \end{cases} \tag{1}$$

where N_z, N_θ, $N_{z\theta}$, $N_{\theta z}$, M_z, M_θ, $M_{z\theta}$, $M_{\theta z}$ are the internal force components. p is sound pressure in flow field. u, v, w denote shell displacements in the axial, circumferential and radial directions, respectively. t is the time.

The internal force components can be obtained from the integrals of internal stress components (Flügge, 1973)