Ocean Engineering 136 (2017) 168–177

Contents lists available at ScienceDirect

Ocean Engineering

journal homepage: www.elsevier.com/locate/oceaneng

Hydrodynamic loads acting on a circular porous plate horizontally submerged in waves

Fenfang Zhao, Tongzheng Zhang, Rong Wan, Liuyi Huang, Xinxin Wang*, Weiguang Bao

Fisheries College, Ocean University of China, Qingdao 266003, China

A R T I C L E I N F O

Keywords:
Porous plate
Eigenfunction
Porosity
Haskind relation

A B S T R A C T

The problem of a circular porous plate horizontally submerged in water is investigated within the context of linear potential theory. The porous plate is either fixed in monochromic waves or forced to oscillate in the heave or pitch mode in still water. Darcy's law is applied to give the boundary conditions on the porous plates. The boundary value problems are solved semi-analytically by means of the eigenfunction-expansion for multiple regions. Two approaches that have commonly been adopted in previous studies are both used in the present study. The hydrodynamic forces acting on the plates are evaluated. Comparisons are made between the results of these two methods and good agreement is observed. The effects of wave parameters, the submerged depth and the porosity of plates are discussed. The Haskind relation is examined and confirmed for the porous plate as well.

1. Introduction

Along with the expansion of ocean engineering to deeper and deeper sea areas, the requirements of effective and inexpensive wave barriers have been increased to protect devices moored in open seas, such as sea cages used in aquaculture. Porous materials, which are effective at dissipating wave energy, have attracted substantial research interest. During recent decades, many studies, both theoretical and experimental, have investigated the hydrodynamic performance of breakwaters made of porous materials and evaluated the wave loads acting on the barriers.

Here, only a few examples are discussed from among the plethora of previous studies. A porous wave-maker theory was proposed and developed by Chwang and Li (1983), and by Chwang (1983), to investigate the effect of porosity on free-gravity waves based on the linear wave theory and Darcy's law. The interaction between a submerged porous disk and ambient wave field was investigated by means of the matched eigenfunction expansion (Chwang and Wu, 1994; Cho and Kim, 2000; Cho and Kim, 2008; Bao et al., 2009; Zhao et al., 2010; Zhao et al., 2011; Cho and Kim, 2013). In these previous studies, the eigenvalues are generally complex numbers. In a different approach, instead of applying Darcy's law, Molin and his research group suggested a quadratic relationship between the pressure difference and the traversing velocity across the porous plate, the latter of which is specified by an eigenfunction expansion to avoid complex eigenvalues (Molin and Legras, 1990; Molin and Nielsen, 2004; Molin et al., 2007). This expansion of the traversing velocity is also applied in other studies, such as those by Liu et al. (2008) and Liu and Li (2011). Cho et al. (2013) extended the method to study dual submerged horizontal porous plates and their analytical results are validated by a series of experiments.

As discussed above, two approaches were mainly adopted in the previous works. In the first approach, a ‘complex wave number’ is sought, and the corresponding eigenfunctions are then determined. However, in the second approach, the traversing velocity distribution across the porous plate is specified by proper expansions.

To compare these two methods in the present work, the problem of a porous circular plate submerged horizontally in water is studied within the scope of linear potential theory. The plate is either fixed in a monochromic wave field or forced to oscillate in still water in the heave or pitch mode, i.e., both the diffraction and radiation problems are considered. Darcy's law is applied to the porous boundary condition. The problem is solved based on the matched eigenfunction expansion. The fluid domain is divided into two regions and different expansions are searched in these regions. They are matched at the common surface to determine the unknown coefficient in the expansions, and the wave loads acting on the plate are calculated. Both methods mentioned above are used to solve the problem and the computed results are compared. The Haskind relation, i.e., the wave exciting forces calculated from the corresponding radiated waves, is checked for the porous plate as well in the present work.

Following the introduction, the boundary value problem is pre-

* Corresponding author.
E-mail address: wxinxin@ouc.edu.cn (X. Wang).

http://dx.doi.org/10.1016/j.oceaneng.2017.03.026
Received 3 August 2015; Received in revised form 3 January 2017; Accepted 13 March 2017

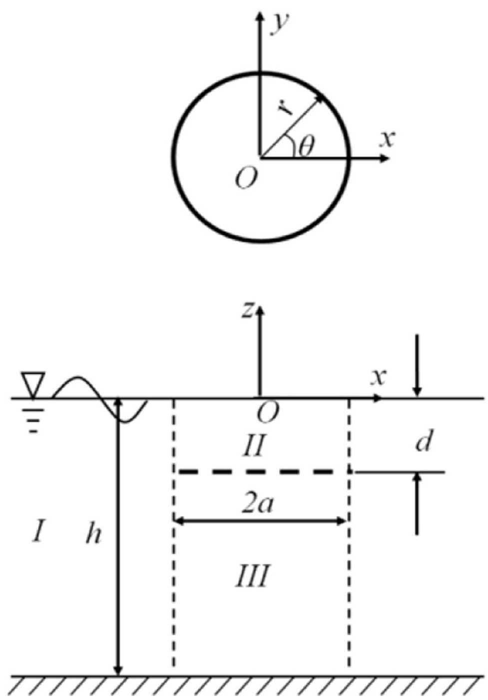

Fig. 1. Definition of the coordinate system and the division of the fluid domain for a submerged horizontal circulate plate.

sented. Approaches to solve the problem are then discussed in the next section. The calculation of wave loads, including the added mass and damping, follows. Some computed results are demonstrated and discussed. A brief summary presents the paper's conclusions.

2. The boundary value problem

A circular plate with a radius a is assumed to be horizontally submerged in water with a depth of h at a depth of d beneath the water surface. The plate is made of a porous material with fine holes. The plate is either fixed in a train of regular waves or is oscillating in the heave or pitch mode in still water. It is noted that the forced oscillation in the surge, sway or yaw mode is trivial for a thin plate, and the roll mode may be considered similarly to the pitch mode due to the symmetry. To evaluate the hydrodynamic forces acting on the plate, a cylindrical coordinate system is adopted to describe the problem. Its horizontal plane, i.e., the r–θ plane, coincides with the still water surface, whereas the z-axis goes through the center of the circular plate and points vertically upwards (see Fig. 1).

The viscosity is assumed negligible, and the motion amplitude is assumed small, so the problem will be solved within the scope of linear potential theory. Further, assuming a harmonic motion with a frequency ω, the velocity potential $\Phi(r, \theta, z, t)$ can be expressed by the product of the harmonic exponential function of the time variable and a field potential $\phi(r, \theta, z)$, depending only on the position, such as:

$$\Phi(r, \theta, z, t) = \mathrm{Re}\{\phi(r, \theta, z)e^{-i\omega t}\}. \tag{1}$$

The latter is further decomposed into the diffraction and radiation potentials, i.e.

$$\phi(r, \theta, z) = \frac{gA}{i\omega}\phi_D(r, \theta, z) + \sum_{s=3,5} -i\omega\xi_s\phi_s(r, \theta, z), \tag{2}$$

where g denotes the gravitational acceleration, ω is the angular frequency and A is the amplitude of the incident waves, and ξ_s is the amplitude of the forced oscillation in the heave (s=3) and pitch (s=5) modes, respectively. The diffraction potential consists of the incident wave potential ϕ_0 and the scattering potential ϕ_7, i.e., $\phi_D = \phi_0 + \phi_7$. The incident wave potential ϕ_0 is known and is written as follows:

$$\phi_0 = \frac{\cosh k_0(z+h)}{\cosh k_0 h}\exp[ik_0 r\cos(\theta-\beta)] = \frac{\cosh k_0(z+h)}{\cosh k_0 h}\sum_{n=0}^{\infty}\varepsilon_n i^n J_n(k_0 r)\cos(\theta-\beta), \quad \varepsilon_n = \begin{cases}1 & (n=0)\\ 2 & (n\ge 1)\end{cases}, \tag{3}$$

where β indicates the incident wave angle, and, due to the symmetry of the circular plate of the present study, it is set to β=0 hereafter with no loss of generality.

All the potentials are governed by the Laplace equation in the fluid domain. In addition, the linearized free surface condition is imposed at the mean water surface, and there is an impermeable condition at the sea bottom. The Sommerfeld radiation condition is satisfied by the scattering and radiation potentials (s=3, 5, 7) in the far field. On the body surface, i.e., on the circular porous plate, Darcy's law is applied with the assumption of fine pores. The normal velocity is taken to be continuous through the porous plate and its magnitude is proportional to the pressure difference between the two sides of the plate. In summary, the boundary value problem consists of the following equations:

$$\nabla^2\phi_s = 0, \quad (-h < z < 0); \tag{4a}$$

$$\frac{\partial\phi_s}{\partial z} - \nu\phi_s = 0, \quad \left(z = 0,\ \nu = \frac{\omega^2}{g}\right); \tag{4b}$$

$$\frac{\partial\phi_s}{\partial z} = 0, \quad (z = -h); \tag{4c}$$

$$\left.\frac{\partial\phi_s}{\partial z}\right|_{(z=-d+0)} = \left.\frac{\partial\phi_s}{\partial z}\right|_{(z=-d-0)} = n_s + i\sigma[\phi_s|_{(z=-d-0)} - \phi_s|_{(z=-d+0)}] \quad (r < a), \quad n_s = \begin{cases}1 & (s=3)\\ -r\cos\theta & (s=5)\\ 0 & (s=D)\end{cases}; \tag{4d}$$

$$\lim_{r\to\infty}\sqrt{r}\left[\frac{\partial\phi_s}{\partial r} - ik_0\phi_s\right] = 0, \quad (s = 3,\ 5\mathrm{or}7), \tag{4e}$$

where k_0 is the wave number that satisfies the dispersion relation: $k_0 \tanh k_0h=\nu=\omega^2/g$.

In the porous boundary condition (4d), the quantity z=−d+0 (or −d−0) indicates the location just above (or beneath) z=-d. In the special case of d=0, i.e., when the porous plate coincides with the water surface, it is difficult to define a potential in the region above the plate. To overcome this obstacle, it is assumed that the linear free surface condition (4b) is still valid at z=0, and then the potential can be represented by its vertical derivative, which is continuous across the plate. The relation at the porous boundary is reduced to the following form:

$$\left(1 + \frac{i\sigma}{\nu}\right)\frac{\partial\phi}{\partial z} = i\sigma\phi, \quad (z = -d = 0,\ r < a). \tag{5}$$

The porous effect parameter σ in Eq. (4d) is a real and positive number, which is normalized by the wave number k_0 to obtain the so-called porosity parameter $b=2\pi\sigma/k_0$. For the value b=0, the porous plate becomes a solid plate. At the other extreme, i.e.$b \to \infty$, the porous plate is transparent or no longer exists. As a result of a series of systematic experimental investigations in the previous work (Zhao et al., 2011), an empirical formula is deduced to relate the porosity parameter b to the puncture ratio τ of the plate:

$$b = \frac{(17.80/\varepsilon + 143.2)\tau^2}{1 + 1.06\tau}, \tag{6}$$

where the puncture ratio τ is defined as the ratio of the total area of the holes to the whole area of the plate, and the wave slope is given by $\varepsilon=Ak_0$.