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Hydro-mechanical Behaviors of the Three-dimensional Consolidation of Multi-layered Soils with Compressible Constituents

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ABSTRACT

Compressibility and permeability are two major coupled factors for the engineering properties of soils. Such hydro-mechanical behaviors of soil greatly rely on the composition and the structure of soil which is often expressed in terms of porosity. In this paper, the effect of soil porosity on the soil mechanisms is studied in the research of Biot's consolidation for soft soil. Assuming a constant porosity in a finite layer, the drained bulk modulus and the permeability are porosity-dependent, and the fluid and the soil skeleton of highly saturated soil are assumed to be compressible. The expressions of displacement, stress and pore pressure are obtained with the aid of Laplace-Hankel transforms for the axisymmetric problem. The continuity of multilayered soil yields the global stiffness matrix in the transformed domain. Meanwhile, by introducing the stiffness matrix of the soil base, the compressible and permeable boundary conditions are more acceptable in practical engineering and the global stiffness matrix is then solved in the transformed domain. Taking the inverse of the Laplace-Hankel transforms gives the real answers for Biot's consolidation. The results show that the increase in porosity leads to a higher compressibility and permeability of soil, and consequently, causes a higher settlement and accelerates the process of consolidation. However, the assumption of the constant porosity of soil has some limitations due to ignoring the nonlinear development of soil structure during consolidation and the further works about the variation modes of porosity need to be done for a better understanding of soil consolidation.

1. Introduction

Highly saturated seabed sediment consists of solid particles of various sizes with interconnected void spaces that permit water flow after a compressive stress from a vertical loading. Under the complicated environment of seawater, the soil conditions are often complicated due to currents, biological working and chemical alteration, which results in a sensitive soil structure. How much the compression of such soils will be and at what rate the compression will occur have drawn great interest by numerous investigations in geotechnical engineering (Fox et al., 2005; Gourvenec and Randolph, 2010; Liu et al., 2013). Developed originally from the one-dimensional consolidation (Terzaghi, 1943), Biot extended the investigation of three-dimensional consolidation which is regarded as the most reasonable model (Biot, 1941). Henceforth, numerical and analytical efforts have been made to solve the equations governing Biot's consolidation. As a useful numerical means, the transforms technology has been widely employed in the analysis of soil behavior, including the elastic semi-infinite soil medium (McNamee and Gibson, 1960; Schiffman and Fungaroli, 1965) and the finite soil layer confined by a pervious or impervious base (Gibson et al., 1970; Booker, 1974; Ai and Wang, 2008). Meanwhile, the natural foundation usually involves stratification requiring the investigation of multilayered soils. Therefore, the finite layer method by Fourier transform (Booker and Small, 1982a; Yue, 2015) and the Laplace-Fourier transforms (Booker and Small, 1987; Vardoulakis and Harnpattanapanich, 1986), the transfer matrix method (Wang and Fang, 2003; Ai et al., 2010) and the analytical layer-element method (Ai et al., 2011) were further developed to obtain the solutions for Biot's consolidation of multilayered soils.

In the aforementioned results, the fluid and solid constituents were assumed to be incompressible for simplicity. In fact, including a small amount of air in a highly saturated soil can reduce the bulk modulus of water dramatically (Verruijt, 1969), influencing the consolidation process and the pore pressure distribution (Cheng and Liggett, 1984; Booker and Cater, 1987) and causing a higher initial settlement (Chen, 2004). A further step which took into account both the compressibility of solid and fluid had been reported to study the time behaviors of a multilayered medium (Senjuntichai and Rajapakse, 1995) employing

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Nomenclature

a_c	grain contact area per unit area of plane;
C_f	compressibility of fluid, $C_f = 1/K_f = \frac{S_r}{K_w} + \frac{1-S_r}{P_{w0}}$;
C_g	compressibility of solid grain;
C_m	compressibility of soil;
C_s	compressibility of soil skeleton;
D, F	displacement function;
E	Young's modulus of soil;
G	shear modulus of soil;
h_i	thickness of the ith layer;
H_i	distances from the surface to the bottom of the ith layer;
$J_m(r\xi)$	first kind of mth-order Bessel function;
K	drained bulk modulus of soil;
K_b, n_b, k_b	drained bulk modulus, porosity and permeability of soil base;
K_s	bulk modulus of porous matrix ($K_s = 1/C_s$);
K_u	undrained compression modulus, $K_u = K + \frac{\alpha^2}{nC_f + (\alpha - n)C_s}$;
K_w	bulk modulus of air free water;
k	permeability;
n	porosity;
P_{w0}	total static pore water pressure,
q	external pressure;
Q	total flow during a given time;
S_g	specific surface area of soil grains, $k_0 = 1/(5S_g^2)$;
S_r	saturation degree
u_r, u_z	displacements at r and z direction;
α	fraction of the pore pressure that gives the effective stress
γ_w	unit weight of water;
ε	volumetric strain;
θ	friction angle of solid phase;
θ'	effective angle for shear resistance;
ν	Poisson's ratio of soil;
ν_u	undrained Poisson's ratio of soil, $\nu_u = \frac{3K_u - 2G}{2(3K_u + G)}$;
σ_v'	effective mean stress;
σ_{rr}, σ_{zz}	total stresses in r and z directions, respectively;
σ_{rz}, σ_{zr}	shear stresses;
τ	time factor.

the analytical model (Rice and Cleary, 1976). Since then, the consolidation of soil with compressible fluid and solid constituents had been extensively investigated (Chen et al., 2005; Singh et al., 2009; Rani et al., 2011; Ai and Hu, 2015).

Reviewing the aforementioned achievement, emphasis had been placed on obtaining the feasible and efficient solutions to the coupled governing equations of Biot's consolidation, while the hydro-mechanical behaviors of soil were greatly simplified. For instance, the permeability of soil, k, is assumed constant, together with the compression parameters, such as the drained bulk modulus K. Nonetheless, the permeability (Carman, 1956; Chapuis, 2012) and the deformation modulus (Kupkova, 1993; Lu et al., 1999) are porosity-dependent in porous materials suggesting the hydro-mechanical properties of soils (Rutqvist and Stephansson, 2003; Bandara and Soga, 2015). As for a specific soil, the value of the porosity can range from 0.2 to 0.6 (Lambe and Whitman, 1969), depending on both the intrinsic and external factors: (1) particle sizes vary and smaller particles can occupy pore spaces between larger particles, producing a tendency toward lower porosities and higher densities; (2) the sedimentation process of soil is affected by the varying surroundings. The porosity of a highly saturated soil determines the strength properties and permeability properties that, in turn, control such important processes such as the consolidation rate, and generation of excess pore pressures. Consequently, in this paper, the porosity will be considered addressing the following two aspects:

(1) *Drained bulk modulus*. Deformation properties of porous media have attracted attention in geomechanics and energy resource exploration for decades (Zimmerman et al., 1986), and the relationship between the bulk modulus and porosity has been proposed (Zimmerman, 1991). The drained bulk modulus is defined as the relationship between volumetric deformation and stress:

$$K = \frac{E}{3(1 - 2\nu)} = -\frac{d\sigma_v'}{d\varepsilon} \tag{1}$$

E and ν are Young's modulus and Poisson's ratio of the porous media, respectively; σ_v' is the effective mean stress and ε is the volumetric strain. The relationship between the drained bulk modulus and the porosity can be expressed as (Han and Dusseault, 2003; Luo and Stevens, 1999)

$$K = \frac{1 - n}{1 + \kappa n}K_s \tag{2}$$

where n is the porosity, K_s is the bulk modulus of porous matrix ($K_s = 1/C_s$, C_s is the compressibility of porous matrix), and $\kappa = \frac{1 - \nu_s}{2(1 - 2\nu_s)}$, a factor subject to Poisson's ratio and morphology of porous matrix.

(2) *Permeability*. The permeability k is a function of the pore geometry. Based on the flow of water through the channels in soil, a theoretical connection between permeability and porosity exists in the literatures (Carman, 1956; Kozeny, 1927), and Kozeny-Carman model can be expressed as:

$$k = \frac{1}{5S_g^2}\frac{n^3}{(1 - n)^2} = k_0\frac{n^3}{(1 - n)^2} \tag{3}$$

where S_g is the specific surface area of soil grains (Sanzeni et al., 2013) and $k_0 = 1/(5S_g^2)$.

In fact, obtaining a consistent relationship of porosity with permeability for soil is not easy although the Kozeny-Carman equation can work well for describing sand and silt (Carrier, 2003; Chapuis, 2004; Deng et al., 2011). However, Eq. (3) provides a physical-based consideration of permeability-porosity relationship for an average condition of a porous medium (Wang et al., 2009).

From the aforementioned reviews, the porosity plays an important role in the engineering properties of soil. However, the hydro-mechanical coupling effect is rarely considered in the research of Biot's consolidation for soft soil. Hence, the aim of this paper is to investigate the hydro-mechanical behaviors of the multilayered soils with compressible constituents. The drained bulk modulus and permeability of soil relating to the porosity are provided by Eq. (2) and Eq. (3), respectively. The porosity is assumed to be constant during the consolidation for simplification in this analytical model, and the limitations will be presented in the discussion. Derived from Biot's governing equations, the expressions of displacement, stress and pore pressure are obtained from the two displacement functions subject to an axisymmetric loading. Adopting Laplace-Hankel transforms, the global stiffness matrix can be obtained by making use of the continuity of the interfaces between the adjacent layers and can be solved by introducing the boundary conditions. Then, by taking the inversion of the Laplace-Hankel transforms, the real answers for Biot's consolidation are obtained. To verify the accuracy of present method, comparisons with existing results are carried out. Moreover, a series of parametric studies are conducted to analyze the hydro-mechanical behaviors of soils with compressible constituents.