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Variational optimization of hydrokinetic turbines and propellers operating in a non-uniform flow field

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ABSTRACT

The purpose of this paper is to provide a correction for the optimum distribution of circulation of propellers and turbines, though its effect is more noticeable in turbines. The correction is related to the fact that, when looking for the optimum, the induced speeds are derived with respect to the circulation, but the component associated to the change in direction of the free vortices has been traditionally neglected. The main result is thus a new expression for the derivatives of the induced velocities obtained by including the effect produced by a change in wake pitch due to an infinitesimal change of the circulation. The net contribution of this paper is an improved method for hydrokinetic turbine optimization, built upon the propeller vortex lattice framework, which is consistent with prior optimization methods for propellers but improves upon prior optimization methods for turbines. The result extends the theory to non-uniform axisymmetric flows. Further analysis of Lerbs/Munk original theory is provided. Example propeller and turbine designs demonstrate the utility of the Present method.

1. Introduction

The paper is motivated on the fact that the optimization methods of classic lifting-line theory have traditionally failed when used for turbine design. This is well known and explained, for instance, in Epps and Kimball (2013a). The objective of this paper is thus to provide a correction for the optimum distribution of circulation of propellers and turbines. This correction is related to the fact that, when looking for the optimum, the induced speeds are derived with respect to the circulation. However, the component of this derivative associated to the change in direction of the free vortices has been traditionally either neglected or avoided due to its relative complexity.

If it is avoided is by applying instead Munk's theorem, as done in Betz (1919) and Lerbs (1952). Lerbs (1952) used this assumption proposed in Munk (1923) to obtain his criterion for the optimum distribution of circulation for a propeller. Unfortunately this same assumption applied to turbines results in a non-optimum outcome.

In this paper, we will show this issue and solve the same problem tackled by Lerbs without using Munk's assumption. We will do so by calculating the neglected component, which could be assessed numerically once some convergence issues are solved, as mentioned in Menéndez Arán and Kinnas (2012). However, an analytical expression is possible for the case of an infinite number of blades. We will obtain this expression in Section 2. In Section 3 we will use this expression in the numerical design of a specific turbine and propeller, comparing the results with older methods. Section 4 details the mathematical manipulations needed to obtain the expression for the optimum. The derived expression will provide a very small correction to propellers, where Lerbs' criterion will be recovered in moderately-loaded conditions, but a more significant correction to turbines.

For the numerical comparisons given in Section 3 we will use OpenProp (see the web (Epps and Kimball 2013b)) to compare several optimization criteria, where some are based on Munk's assumption and others are not. OpenProp is a free software that applies the lifting-line theory to the design of propellers and turbines. A summary is given in Epps (2010) and the most recent information is given in Epps and Kimball (2013a).

1.1. Vortex lattice lifting-line theory

Reference (Kerwin and Hadler, 2010) explains the theory of lifting line. Many authors have contributed to its development: Lanchester set the bases of the theory in Lanchester (1907). Prandtl developed it for propellers in Prandtl (1921) using an approximate formulation for the

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Nomenclature

r	radius
$\beta(r)$	pitch angle
$\beta_i(r)$	pitch angle including the velocities induced at the screw plane
β_d	pitch angle including the velocities induced downstream
$w_x(r)$	wake coefficient at a radius r
V_S	design speed (ship or current speed)
$V_a(r)$	inflow axial speed
$V_t(r)$	inflow tangential speed
ω	rotational speed of the propeller or turbine
$u_a^*(r)$	axial induced speed at r in a lifting line
$u_t^*(r)$	tangential induced speed at r in a lifting line
$V^*(r)$	local speed including the induced velocities at a radius r in a lifting line
$\tilde{u}_a^*$	mean axial induced velocity at the screw plane
ρ	fluid density
Z	number of blades or lifting lines
r_h	hub radius
R	maximum radius of the screw
D	diameter of the screw: (D =2 R)
A	area swept by the screw: (A= πR^2)
$\Gamma(r)$	distribution of circulation along the lifting line
$c(r)$	chord length of the blade at a radius r
C_D	drag coefficient of the foil section
T	total thrust of the propeller (if bigger than 0) or turbine (if smaller than 0)
Q	total torque of the propeller or turbine
C_T	thrust coefficient, $C_T = T/(0.5\rho\ \pi R^2 V_S^2)$
J	advance ratio, $J = (\pi\ V_S)/(\omega R)$
η	efficiency
η_O	optimum efficiency
η^*	efficiency related to a change of circulation, equal to $\frac{T'\ V_S}{Q'\ \omega}$ in a propeller.
r_c	control position at the lifting line where the induced velocities and forces are calculated
r_v	radial position at the lifting line where a free vortex is shed
r_g	radial position at the lifting line around which a horseshoe vortex with radius r_{v+1} and r_v is shed
r_g	radial position at the lifting line around which a horseshoe vortex with radius r_{v+1} and r_v is shed
$u_{ad}(r_d)$, $u_{td}(r_d)$	total axial and tangential velocities induced downstream at r_d
$\bar{u}_a(r_c, r_v)$, $\bar{u}_t(r_c, r_v)$	axial and tangential velocities induced at r_c (in the lifting line) by Z free vortices of unit circulation shed from r_v in each lifting line
$\bar{u}_a^*(r_c, r_g)$, $\bar{u}_t^*(r_c, r_g)$	axial and tangential velocities induced at r_c (in the lifting line) by Z horseshoe vortices of unit circulation shed around radius r_g in each lifting line

induced velocities. Goldstein improved the calculation of the induced velocities for the case of uniform inflow in Goldstein (1929). Then, making the best of Glauert's sine series method (Glauert, 1947) (though the 1st edition dates from 1926), Lerbs and Wrench established the application of lifting-line theory for propellers in axisymmetric non-uniform inflow respectively in Lerbs (1952) and Wrench (1957). Kerwin has worked extensively in the application of vortex-lattice methods to the theory, as in Kerwin (1961).

The representation of a screw with the lifting-line method uses Z lines, one for each blade of the screw (Fig. 1). Bound vortices spread out along each line with a distribution of circulation $\Gamma(r)$ that begins at r_h, which is the external radius of the hub, and ends at R, which is the radius corresponding to the blade tip. Free vortices of circulation are shed from the blades of the propeller/turbine, creating helicoidal surfaces downstream.

The loads on the screw can be readily evaluated by invoking the Kutta-Joukowski theorem, where lift is given by $\rho\ V^*\Gamma$, and by ignoring viscous forces. ρ is the fluid density, V^* is the total speed at one point in the lifting line (considering the effect from the induced velocities), and Γ is the circulation around this same point. The thrust T and torque Q produced by the screw are:

$$T = \rho\ Z \sum_{m=1}^{M} (\omega r(m) + u_t^*(m))\ \Gamma(m)\ \Delta r(m) \tag{1.1}$$

$$Q = \rho Z \sum_{m=1}^{M} (V_a(m) + u_a^*(m))\ \Gamma(m)\ r(m)\ \Delta r(m) \tag{1.2}$$

with all *(m)* summation terms evaluated at $r = r_c(m)$, which are known as control points. $u_a^*(m)$ and $u_t^*(m)$ are respectively the axial and tangential induced velocities in the lifting line at $r_c(m)$.

For extension to non-zero swirl inflow $V_t(r)$, replace $\omega\ r$ with $\omega\ r + V_t(r)$ throughout. Viscous forces can be handled by including a viscous drag term $\frac{1}{2}\rho(V^*)^2 c\ C_D$ in each of these summations (Kerwin and Hadler, 2010), but this is not presented herein for clarity. In this expression c is the chord length of the blade section, and C_D is the drag coefficient of the section.

Fig. 2 shows a two-dimensional diagram of speeds and forces in a propeller blade element. Fig. 3 shows the same for a turbine blade element. We will consider the screw is the only element producing vorticity to the flow. This is, there is no nozzle. We will also consider the screw is working in a steady regime and in an incompressible fluid.

The axial (u_a^*) and tangential (u_t^*) induced velocities at an arbitrary point $r_c(m)$ of the lifting line can be obtained from the contributions of all free vortices:

$$u_a^*(m) = \sum_{n=1}^{M} \bar{u}_a^*(m, n)\ \Gamma(n)$$
$$u_t^*(m) = \sum_{n=1}^{M} \bar{u}_t^*(m, n)\ \Gamma(n) \tag{1.3}$$

where $\bar{u}_a^*(m, n)$ and $\bar{u}_t^*(m, n)$ are the axial and tangential velocities induced at $r_c(m)$ on the key blade by Z unit-strength 'horseshoe vortices', each surrounding the nth panel of each blade (where a horseshoe vortex consists of a segment of the lifting line and the trailing vortex filaments shed from its endpoints). The horseshoe influence functions ($\bar{u}_a^*$ and $\bar{u}_t^*$) are in general computed via the Biot-Savart's law, but for a purely helical wake, they can be evaluated analytically using the formulas in Lerbs (1952) and Wrench (1957), as detailed in Chapter 2.

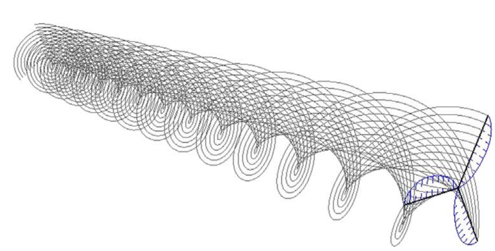

Fig. 1. Representation of a screw using 1 lifting line (in bolt black) for each blade, a free-vortex surface (in grey) shedding from each line, and the distribution of circulation (in blue) along each lifting line. In this picture r_h =0 and Z =3. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)