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Band gaps for Bloch waves over an infinite array of trapezoidal bars and triangular bars in shallow water

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A B S T R A C T

The problem of shallow-water wave propagation over an infinite array of periodic trapezoidal bars and triangular bars is studied. By employing the linear shallow-water wave theory and the well-known Bloch theorem, the eigenvalue problem in terms of the wave number for given wave frequency, water depth, and geometric properties of the bars is formed. A closed-form solution of the eigenvalue problem is derived which breaks the restriction of piecewise constant water depth required in almost all of the previous analytic solutions. The solution is verified against the existing solution for the special case of rectangular bars. Based on the present solution, gap maps in various cases are plotted which exactly show the distribution of band gaps for wave propagation over both trapezoidal bars and triangular bars, and the influence of the given wave frequency and the geometric parameters of bars such as height and width on the occurrence of band gaps is analyzed. By using the gap maps presented in the paper, the condition under which the waves can be completely blocked by an infinite array of trapezoidal bars and triangular bars can be easily and exactly determined.

1. Introduction

The scattering of ocean surface waves by a finite array of periodic structures is of great practical importance in coastal and ocean engineering since finite arrays of periodic structures are very common in practice, such as rippled bottoms (Davies and Heathershaw, 1984; Mei, 1985; Naciri and Mei, 1988; Chamberlain and Porter, 1995; Alam, 2012), periodic artificial sandbars (Mei et al., 1988; Jeon and Cho, 2006; Liu et al., 2015), periodic rigid vegetation (Ozeren et al., 2014; Liu et al., 2015; Tang et al., 2015), vertical cylinder arrays (Linton and Evans, 1990; Hu et al., 2004; Jeong et al., 2004; Farhat et al., 2008), periodic floating buoy arrays (Garnaud and Mei, 2010), periodic flexible floating plates or floating membranes (Manamn and Kaligatla, 2012) and periodic resonator arrays (Hu et al., 2011, 2013).

When ocean surface waves are scattered by a finite array of periodic structures, they can be greatly modulated by the introduced periodicity due to multiple Bragg scatterings, Bragg resonant reflection occurs when the periodic length of the structure is an integer multiple of the half-wavelength of the incident waves. It has been shown by many studies (Davies and Heathershaw, 1984; Hsu et al., 2002; Cho et al., 2004; Jeon and Cho, 2006; Liu et al., 2012) that the Bragg resonance increases with the increase of the size of the finite array. This raises a question that if the size of the finite array becomes very large, can the periodic structures completely block the incident waves?

The answer to the above question relies on the solving of the eigenvalue problem which investigates the existence of unforced waves over an infinite array of periodic structures. According to the Bloch theorem, for waves propagating in a periodic medium, they will be expressed into a product of a plane wave and a periodic function which has the periodicity of the medium. Waves with such a property are called Bloch waves. Further, according to the band theory, the propagation of Bloch waves is characterized by the related band structures, between bands there may exist a band gap (or stop band) within which wave propagation is absolutely forbidden. Therefore, by solving the eigenvalue problem of Bloch waves for infinite periodic topography and finding out the band gap (if it exists), then we know the incident waves at frequencies within the band gap can be totally stopped or reflected when the size of the finite array becomes very large.

Due to the theoretical significance of the eigenvalue problem of Bloch waves, the calculations on the band structures and the possibility of the existence of band gaps for water wave propagation over infinite array of periodic structures have received considerable attention since 1990s, see An and Ye (2004), Huang et al. (2005) and Tang et al. (2006) for one-dimensional structures and Hu et al. (2003), Torres et al. (2003), Ye (2003), Chen et al. (2004) and Jeong et al. (2004) for two-dimensional structures. However, most of these previous investigations are restricted to vertical structures with piecewise constant depths, such as rectangular steps (An and Ye, 2004; Huang et al., 2005; Wang et al., 2007), rectangular trenches (Huang et al., 2005), water

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piercing cylinders (Hu et al., 2004; Jeong et al., 2004; Yang et al., 2009), drilled cylindrical holes (Torres et al., 1999; Hu et al., 2003; Torres et al., 2003; Shen et al., 2005a,b) and submerged cylinders (Ye, 2003; Chen et al., 2004). Only a few periodic structures with continuous depth variation has been studied in recent years, such as an infinite periodic arrays of submerged horizontal circular cylinders in deep water (Linton, 2011) and a channel with a gently corrugated bottom (Piat et al., 2015).

There is no doubt that the above restriction will narrow down the application range of the band structure theory since most of the coastal periodic bottom topographies, e.g. sandbars and ripples (Davies and Heathershaw, 1984; Mei, 1985; Dalrymple and Kirby, 1986; Kirby, 1986; Liu et al., 2012) and periodic structures, e.g. artificial sandbars or Bragg breakwaters (Mei et al., 1988; Cho et al., 2004; Jeon and Cho, 2006; Hsu et al., 2007; Tsai et al., 2011; Liu et al., 2015) are related to continuously varying water depths.

The main difficulty in dealing with continuously varying bottoms or structures comes from two facts that, first, if the Laplace equation related to the infinitesimal amplitude water wave theory (Mei, 1989; Dingemans, 1997) is adopted as the governing equation, then the boundary condition at the irregular bottom is quite complex; second, if a depth-averaged equation such as linear shallow-water equation (Mei, 1989; Dingemans, 1997) or the mild-slope type equation (Chamberlain and Porter, 1995; Liu and Zhou, 2014) is adopted, then a differential equation with variable coefficients or even with implicit variable coefficients needs to be solved.

In this paper, we study the band structure of shallow-water waves over an infinite array of uniformly spaced trapezoidal bars and triangular bars, where the water depths in both the front and the back slope regions vary continuously.

2. An infinite array of trapezoidal bars

At first, we study trapezoidal bars which are parallel to the coastline and located at a horizontal bed, see Fig. 1 for the sketch of the problem, where h_0 is the global water depth, h_1 is the bar submergence, w_b is the width of the bar bottom, w_t is the width of the top plane of the bar and d is the distance between two adjacent bars, which is the periodicity of the bottom with bars.

According to the Bloch theorem, the waves will exhibit the Bloch state and the surface elevation $\eta(x)$ can be expressed as the product of a periodic function $B(x)$ and a plane wave e^{iKx} as follows:

$$\eta(x) = B(x)e^{iKx}, \tag{1}$$

where K is the Bloch wave number, and $B(x)$ has the same periodicity, d, of the bottom, i.e., $B(x + d) = B(x)$. It is clear that the Bloch theorem greatly simplifies the study of the propagation of surface waves over the global periodic bottom into the study within one period. Specifically, by

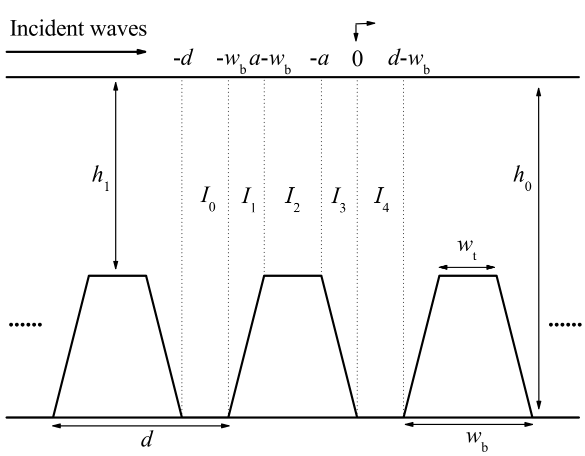

Fig. 1. An array of periodic trapezoidal bars (side view).

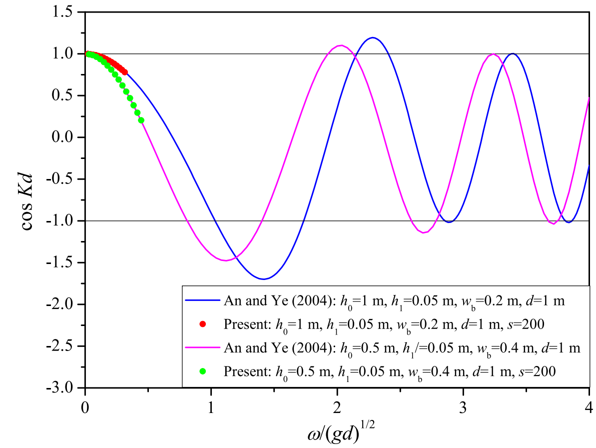

Fig. 2. Comparison between the present solution for trapezoidal bars with s=200 and the solutions for rectangular bars given by An and Ye (2004).

using Eq. (1), we only need find out the expression of $\eta(x)$ within one periodic interval $[-d, 0]$.

According to the linear shallow-water wave theory (Mei, 1989; Dingemans, 1997), the surface elevation $\eta(x)$ satisfies the shallow-water equation as follows:

$$h(x)\eta''(x) + h'(x)\eta'(x) + \frac{\omega^2}{g}\eta(x) = 0, \tag{2}$$

where $h(x)$ is the water depth, ω the angular frequency and g the gravitational acceleration.

For convenience, we divide the interval $[-d, 0]$ into four subintervals: $I_0 = [-d, -w_b]$, $I_1 = [-w_b, a - w_b]$, $I_2 = [a - w_b, -a]$, $I_3 = [-a, 0]$, where $a = (w_b - w_t)/2$, see Fig. 1. Within both I_0 and I_2, the water depths are constants h_0 and h_1, respectively, and Eq. (2) degenerates into the Helmholtz equation, and its general solutions are trivial as follows:

$$\eta(x) = \begin{cases} A_1 e^{ik_0x} + A_2 e^{-ik_0x}, & x \in I_0, \\ A_5 e^{ik_1x} + A_6 e^{-ik_1x}, & x \in I_2, \end{cases} \tag{3}$$

where i is the imaginary unit, $k_0 = \omega/\sqrt{gh_0}$ and $k_1 = \omega/\sqrt{gh_1}$.

In the subinterval I_1, since $h(x) = h_0 - s(x + w_b)$ with $s = (h_0 - h_1)/a = 2(h_0 - h_1)/(w_b - w_t)$ being the slope of the front slope, Eq. (2) becomes

$$[h_0 - s(x + w_b)]\frac{d^2\eta(x)}{dx^2} - s\frac{d\eta(x)}{dx} + \frac{\omega^2}{g}\eta(x) = 0. \tag{4}$$

Following Liu et al. (2013), we introduce transforms as follows:

$$t = \frac{2k_0\sqrt{h_0(h_0 - sx - sw_b)}}{s}, \quad \tilde{\eta}(t) = \eta(x), \tag{5}$$

then Eq. (4) can be transformed into

$$t^2\frac{d^2\tilde{\eta}(t)}{dt^2} + t\frac{d\tilde{\eta}(t)}{dt} + t^2\tilde{\eta}(t) = 0. \tag{6}$$

The general solution is sought to be

$$\tilde{\eta}(t) = A_3J_0(t) + A_4Y_0(t), \tag{7}$$

i.e.,

$$\eta(x) = A_3J_0(2k_0\sqrt{h_0(h_0 - sx - sw_b)}/s) + A_4Y_0(2k_0\sqrt{h_0(h_0 - sx - sw_b)}/s), \tag{8}$$

where J_0 and Y_0 are the Bessel functions of the first and second kinds to order 0, respectively.

Similarly, within the subinterval I_3, the general solution to Eq. (2) is

$$\eta(x) = A_7J_0(2k_0\sqrt{h_0(h_0 + sx)}/s) + A_8Y_0(2k_0\sqrt{h_0(h_0 + sx)}/s). \tag{9}$$