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Optimal collocation of Bragg breakwaters with rectangular bars on sloping seabed for Bragg resonant reflection by long waves☆

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A B S T R A C T

In this paper, we provide a closed-form analytical solution for the reflection of linear long waves propagating over a series of submerged rectangular breakwaters which are located on sloping seabed. It shows that the peak value of reflection coefficient decreases while the slope of seabed increases. And moreover, in view of the present analytical model, the magnitude of the peak Bragg resonant reflection should be affected by the number of breakwaters, dimensionless breakwater submergence and width for the given slope gradient of seabed. Finally, the optimal collocation curves for Bragg resonant reflection are presented for different number of breakwaters. As applications, these optimal collocation curves can be used as quantitative indicators for engineers.

1. Introduction

As we know that when the ocean surface waves propagate from deep water into shallow water, many important physical phenomena, such as shoaling, reflection, refraction, diffraction and so on, may appear due to the change of seabed topographies or underwater structures. More specifically, if the seabed topography or underwater structure is periodic, then the so-called Bragg resonant reflection may happen while the Bragg resonant reflection condition is met. For the extensive studies of Bragg resonant reflection phenomenon over natural periodic sandbars and sand ripples by experiments, theoretical analysis and numerical simulation, ones may refer to Davies and Heathershaw (1984), Heathershaw (1982), Cho and Lee (2000), Davies (1982), Liu et al. (2012), Mei (1985) and Dalrymple and James (1986), Kirby (1986) respectively.

Nevertheless, since the direct application of natural periodic sandbars and sand ripples into coastal engineering technology is impossible. So in 1988, Mei et al. first proposed a very interesting design, that is, we may build a series of small-size, low-height and shore-parallel submerged artificial bars to prevent the drilling platform from the attack of storm waves on the oil fields in the Ekofisk of the North Sea. We can also call these types of submerged artificial bars are Bragg breakwaters since the phenomenon of Bragg resonant reflection. Until now, various shapes of Bragg breakwaters have been suggested and studied into coastal engineering technology. Such as, rectangular, triangle, trapezoid, rectified cosine, parabolic, semi-circular and half-ellipse, etc. For more details, see Refs. Cho et al. (2001), Hsu et al. (2002, 2001, 2007); Kirby and Anton (1986), Wang et al. (2006).

To our knowledge, the studies on the optimal collocation of Bragg breakwaters are still few. In Hsu et al. (2003), in view of extended hyperbolic mild-slope equation, the authors considered the Bragg resonant reflection of forward and oblique incident wave by multiply composite artificial bars. They found that the Bragg resonant reflection can be significantly improved by the way of increasing the number and height of artificial bars. In Chang and Liou (2007), the authors studied the Bragg resonant reflection of trapezoidal artificial bars by the matrix multiplication method in accordance with the long-wave equation. Their conclusions showed that the top plane width and arrangement of trapezoidal were two important parameters affecting the design of multiple composite Bragg breakwaters. Obviously, by observation, we know that the results described above are still more focused on the qualitative analysis than the precise quantification in the design of Bragg breakwaters.

More recently, motivated by the aforementioned contributions, Zeng and Liu et al. in their papers (Zeng et al., 2013; Liu et al., 2014) and Liu et al. (2015) provided different optimal collocation curves for rectangular, triangular, rectified cosine, idealized trapezoidal and parabolic Bragg breakwaters, respectively, located on horizontal seabed by long waves. However, as we know, in reality when the ocean surface waves propagate from the areas of deep water into the near-

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shore, the depth of seabed should be more and more shallow in most cases. Therefore, considering Bragg breakwaters located on the sloping seabed will be more in line with practical problems. Thus this paper can be seen as the continuous work of Zeng et al. (2013), Liu et al. (2014) and Huan-wen et al. (2015).

The paper is structured as follows. In Section 2, we first construct a new model which states that a series of rectangular Bragg breakwaters located on the sloping seabed. And then, based on the technique of matrix multiplication, we obtain an exact analytical solution while the linear long ocean surface waves propagate over the constructed model. In Section 3, to illustrate the positivity of our results, we consider the model validation in three aspects. And moreover, the validation against the Bragg resonant reflection is provided in Section 4. In Section 5, by employing the present analytical solution, several important parameters, such as the number of breakwaters, dimensionless breakwater submergence and width, and so on, which may influence the peak Bragg resonant reflection are discussed in detail. In Section 6, the optimal collocation curves are given for different number of rectangular breakwaters, which may be very important and useful in the fundamental design and construction of Bragg breakwaters in engineering. And finally, for convenience of reader, we post the conclusions of our paper in Section 7.

2. Analytical solution for reflection coefficient

In this section, we shall study a model which states that the linear long waves propagate over a series of submerged rectangular breakwaters which are parallel to the coastline and uniformly located on the sloping seabed, as shown in Fig. 1. Based on the technique of matrix multiplication, we will deliver the analytical approach for the reflection coefficient of the model under consideration.

Assume that the x-axis positively points in wave incident direction and is set on the mean water depth. The original coordinate system is set at the toe of the ascending slope of seabed. From Fig. 1, we derive that the water depth function $h(x)$ can be represented as the piecewise function of

$$h(x)=\begin{cases} h_0, & \text{if } x\le 0,\\ h_1, & \text{if } x\in[x_{j,1},x_{j,2}], \quad j=1,2,\ldots,N,\\ h_0-x\tan\beta, & \text{if } x\in[x_{j,2},x_{j+1,1}], \quad j=1,2,\ldots,N-1,\\ h_2, & \text{if } x\ge x_{N,2},\end{cases} \tag{2.1}$$

where N stands for the number of breakwaters, $\tan\beta$ represents the slope of seabed, and h_p $(p=0, 1, 2)$ denotes the constant depth of water.

In what follows, in accordance with the theory of linear long waves, the water surface elevation $\eta(x)$ should satisfy the long-wave equation

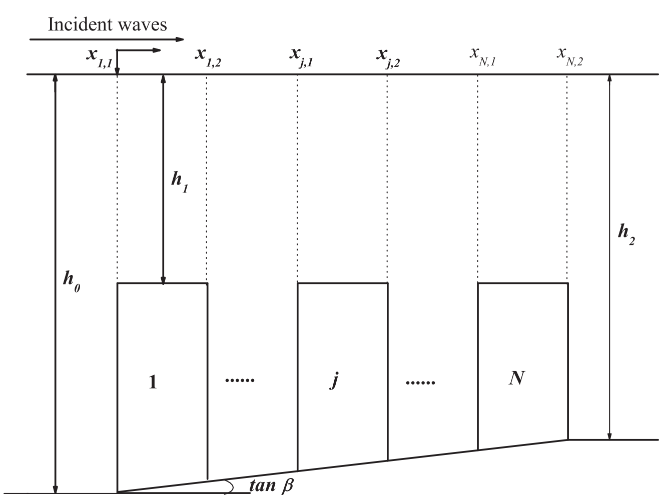

Fig. 1. A sketch of the model.

$$\frac{d}{dx}\left(h\frac{d\eta}{dx}\right)+\frac{\omega^2}{g}\eta=0, \tag{2.2}$$

in which ω and g stand for the angular frequency of the waves and gravitational acceleration, respectively.

Now, let us suppose that the incident waves of unit amplitude comes from the left of Fig. 1, combining with (2.1) and (2.2), we may obtain the solution of long-wave Eq. (2.2) in the sense of constant depths which can be expressed as follows

$$\eta(x)=\begin{cases} e^{ik_0x}+A_Re^{-ik_0x}, & \text{if } x\le 0,\\ A_1^{(j)}e^{ik_1x}+A_2^{(j)}e^{-ik_1x}, & \text{if } x\in[x_{j,1},x_{j,2}], \quad j=1,2,\ldots,N,\\ A_Te^{ik_2x}, & \text{if} x\ge x_{N,2},\end{cases} \tag{2.3}$$

here $i=\sqrt{-1}$, $k_p=\omega/\sqrt{gh_p}$ $(p=0, 1, 2)$ represents the wave number, A_R and A_T are the complex amplitudes of the reflected waves and the transmitted waves, respectively, all $A_R, A_T, A_1^{(j)}, A_2^{(j)}, j=1,2,\ldots,N$ are to be determined. Obviously, the reflection coefficient

$$K_R=A_R$$

and the transmission coefficient

$$K_T=A_T.$$

Next, if $x\in[x_{j,2},x_{j+1,1}]$, $j=1,\ldots,N-1$, inserting (2.1) and (2.2), we have

$$(h_0-x\tan\beta)\frac{d^2\eta}{dx^2}-\tan\beta\frac{d\eta}{dx}+\frac{\omega^2}{g}\eta=0. \tag{2.4}$$

By the auxiliary transforms of

$$t=h_0-x\tan\beta,\qquad \bar\eta(t)=\eta(x), \tag{2.5}$$

we may reformulate equality (2.4) to

$$t\frac{d^2\bar\eta}{dt^2}+\frac{d\bar\eta}{dt}+\sigma\bar\eta=0, \tag{2.6}$$

where $\sigma=\omega^2/(g\tan^2\beta)$. By observing, we see that Eq. (2.6) is a second-order differential equation which can be transformed into a typical Bessel equation of order 0, for more details, refer to Lin and Liu (2005). Therefore, the water surface elevation $\eta(x)$ in the region $x\in[x_{j,2},x_{j+1,1}]$, $j=1,\ldots,N-1$ is written by

$$\eta(x)=A_3^{(j)}J_0(2\sqrt{\sigma(h_0-x\tan\beta)})+A_4^{(j)}Y_0(2\sqrt{\sigma(h_0-x\tan\beta)}). \tag{2.7}$$

Then, for the end of deriving the reflection coefficient of the model, we apply continuity of the surface elevation and the mass conservation cross the common boundaries. Assume that $x=\tilde{x}$ is a common boundary, the continuity of the surface elevation is

$$\eta(x)|_{\tilde{x}^-}=\eta(x)|_{\tilde{x}^+}, \tag{2.8}$$

and in the regime of linear wave theory, which is equivalent to

$$\phi(x,0)|_{\tilde{x}^-}=\phi(x,0)|_{\tilde{x}^+}$$

due to the relationship $\eta(x)=\frac{i\omega}{g}\phi(x,0)$.

In addition, the mass conservation requires that

$$\frac{\mathrm{d}}{\mathrm{d}x}\phi(x,0)|_{\tilde{x}^-}\times\text{the left sectional area}=\frac{\mathrm{d}}{\mathrm{d}x}\phi(x,0)|_{\tilde{x}^+}\times\text{the right sectional area},$$

that is

$$h(\tilde{x}^-)\frac{\mathrm{d}}{\mathrm{d}x}\phi(x,0)|_{\tilde{x}^-}=h(\tilde{x}^+)\frac{\mathrm{d}}{\mathrm{d}x}\phi(x,0)|_{\tilde{x}^+}, \tag{2.9}$$

or

$$h(\tilde{x}^-)\frac{\mathrm{d}}{\mathrm{d}x}\eta(x)|_{\tilde{x}^-}=h(\tilde{x}^+)\frac{\mathrm{d}}{\mathrm{d}x}\eta(x)|_{\tilde{x}^+}, \tag{2.10}$$

which degenerates into