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Free and forced vibration characteristics of submerged finite elliptic cylindrical shell

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ABSTRACT

In this paper, an analytical method to study the free and forced vibration behaviors of a submerged finite elliptic cylindrical shell is proposed. The vibration equations are derived based on Flügge shell theory. Unlike the vibration equations of a circular cylindrical shell, the coefficients of state variables in the vibration equations of the elliptic cylindrical shell are variable with the circumferential curvature, which causes that the vibration equations are rather difficult to solve. To solve this problem, the shell's displacements are expanded in double Fourier series in the view of wave propagation and the circumferential curvature is expanded in single Fourier series. The partial differential equations with variable coefficients are converted into a set of linear equations which couple with each other about the circumferential modal parameters. The fluid around the shell is considered as an ideal acoustic medium and the sound pressure is described by the Helmholtz Equation. Free and forced vibration responses of the submerged finite elliptic cylindrical shell are obtained by solving the coupled equations and are also compared with those of circular cylindrical shell and infinite elliptic cylindrical shell. The present results show good agreements with published results and FEM results. The influences of main parameters of the shell, such as ellipticity parameter, shell thickness ratio, shell length ratio and exciting force's position, on the vibration characteristics are also discussed in detail.

1. Introduction

Elliptic cylindrical shell is a kind of typical structure elements in engineering, and the application of the elliptic cylindrical shell in submarine vehicle, submerged pipeline and other areas is also increasing. Compared with the circular cylindrical shell, the elliptic cylindrical shell has a big advantage on capacity and equipment layout when used in submerged vehicle. With the same cross section area, wetted perimeter area of elliptic cylindrical pipe is smaller than that of square pipe. Besides, even for the originally designed cylindrical shell, due to the manufacturing process, welding deformation, huge pressure, and many other factors, it is difficult to guarantee that the cross section is perfectly circular, and it inevitably appears all kinds of geometric defects including the ellipticity (Gong and He, 2010; Zhang et al., 2011). Therefore, the study of the free and forced vibration characteristics of submerged elliptic cylindrical shell is of great significance.

For thin shell structures, a lot of research focuses on the vibration characteristics of circular cylindrical shell (Zhang et al., 2001; Zhang, 2002; Askari and Jeong, 2010; Shen and Yang, 2014; Chen et al., 2015). However, the studies on the vibration of elliptic cylindrical shell receive scant attention, and the related research mostly concentrates on the free vibration of this type of structure in vacuum. Due to the variable circumferential curvature, the vibration equations of elliptic cylindrical shell are partial differential equations with variable coefficients, which induces the analytical solution to the vibration of elliptic cylindrical shell is much more difficult to obtain than that of the circular cylindrical shell. In order to reduce the difficulty, some approximate methods or numerical methods were developed to study the vibration of elliptic cylindrical shell. Shirakawa and Morita (1982) treated elliptical cross section as two pieces of circular arc linked, and used circular cylindrical shell's equations to establish the vibration equations of elliptic cylindrical shell. The natural frequencies of elliptic cylindrical shell were obtained by considering the boundary continuity conditions of the two pieces of circular arcs, but this method is so complicated that a lot of high order partial differential forms should be dealt with (partial derivative reached the highest 8th order). Klosner (1959), Klosner and Pohle (1958) used the perturbation method to study the free vibration characteristics of elliptic cylindrical shell

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earliest, but this method is only applicable for the shell with ellipticities in a small range. Yamada et al. (1985) used Sanders shell theory and energy-variational principle to establish the vibration equations of elliptic cylindrical shell, and calculated the natural frequencies through Ritz method. Hong and Kim (1995) analyzed the natural frequencies and mode shapes of elliptic cylindrical acoustic cavities by solving the homogeneous wave equation in elliptical cylindrical coordinates, and the sound pressure was expressed in terms of the Mathieu functions. Considering shear deformation and rotary inertia, Suzuki et al. (1996) analyzed the free vibration of laminated composite noncircular thick cylindrical shells based on the thick lamination theory. Ganapathi et al. (2004) used finite element method to deal with the free flexural vibration characteristics of anisotropic laminated angle-ply elliptic cylindrical shells. Grigorenko et al. (2012) employed the spline-collocation method to study the free vibration of thin isotropic elliptic shells with constant thickness. Khalifa (2015b) applied transfer matrix method and Romberg integration method to investigate the influences of the corrugation parameters and material homogeneity on the vibration behaviors of isotropic and orthotropic oval cylindrical shells with sine-shaped hoop. Khalifa (2015a) also studied the free vibration characteristics of a non-homogeneous orthotropic elliptical cylindrical shell resting on the non-uniform Winkler foundation by using transfer matrix method. Tornabene et al. (2015a, b) utilized generalized differential quadrature (GDQ) method to study the free vibration characteristics of the thick and thin elliptic shell structures made of laminated composite materials. Li et al. (2014) proposed a method to the nondestructive prediction of the elastic critical pressure of a submerged elliptic cylindrical shell based on the vibration characteristics, and the natural frequencies were obtained by the transfer matrix method. Some of these mentioned studies are based on numerical methods, which are tedious and inconvenient.

For the fluid-elliptic cylindrical shell coupled system, it is even more difficult to study the vibration characteristics of this type of structure because of the fluid structure coupling effect. Dain (1981) derived the asymptotic formula of the natural frequencies of noncircular cylindrical shells filled with fluid by representing the mode function as a sum of a uniform convergence series and a linear polynomial. Zhu and Weng (1988) analyzed the bending vibration characteristics of an elliptical column partially submerged in water by using the similar method as Dain (1981). Hasheminejad and Aghabeigi (2009) investigated the natural sloshing frequencies of a half-filled non-deformable horizontal cylindrical container of elliptical cross section based on the linear potential theory, and the influences of the container aspect ratio and baffle extension on the sloshing frequencies and transverse oscillation modes were also discussed in detail. Based on the extended homogeneous capacity high precision integration method and the spectrum method of virtual boundary with a complex radius vector, Xiang and Huang (2005) proposed a novel semi-analytical method to study the acoustic radiation of a submerged infinite non-circular cylindrical shell stiffened by longitudinal ribs, and the effects of the number and the size of the longitudinal ribs on the acoustic radiation characteristics of the non-circular cylindrical shell were also analyzed. To overcome the difficulty of analytical methods, numerical simulation methods are also employed in most existing literature. Zhang et al. (2011) used FEM to study the vibro-acoustic characteristics of cylindrical shell with manufacturing error, including initial ellipticity and several other geometry manufacturing errors. Ni and Ai (2007) used FEM/BEM method to analyze the sound radiation characteristics of a composite material elliptic cylindrical shell submerged in water, but the differences of the sound radiation characteristics compared with those of the circular cylindrical shell and the influence of ellipticity parameter on the sound radiation characteristics were not analyzed. Shi et al. (2014) studied the influences of different exciting forces and shell parameters on the vibro-acoustic characteristics of an elliptic cylindrical shell with FEM/BEM method. Recently, Bochkarev and Lekomtsev (2014); Bochkarev et al. (2015, 2016) systemically studied the natural vibrations of non-circular cylindrical shells containing a quiescent compressible ideal fluid based on the developed finite element algorithm, and the sloshing of free surface was also considered for the vibration of partially fluid filled non-circular cylindrical shells.

In this research, based on Flügge (1973) shell theory, the vibration equations of a submerged finite elliptic cylindrical shell are derived with the double Fourier series expansion method and Marguerre's (1951) method to deal with the variable curvature radius of elliptic cross section. Through a series transform, the partial differential equations with variable coefficients are converted into a set of linear equations with constant coefficients. To describe the sound pressure of fluid, angle coordinate in the cylindrical coordinate system of the Helmholtz Equation is changed into a dimensionless arc length coordinate in circumferential direction. The coupled equations are solved to obtain the free and forced vibration responses of elliptic cylindrical shell. The influences of shell ellipticity parameter, shell thickness ratio, shell length ratio and exciting force's position on the vibration characteristics of the finite elliptic cylindrical shell are also analyzed.

2. Mathematical modeling

2.1. Model description

Fig. 1 provides a schematic representation of a thin cylindrical shell with elliptic cross section. It is made of an isotropic material of density ρ, Young's modulus E, Poisson's ratio μ and damping factor η. It is submerged in infinite water domain in which the sound wave propagation velocity is C_f and the fluid density is ρ_f. It is of length L and with constant thickness h. The semi-major and semi-minor axes of elliptic cross section are a and b, respectively.

The shell is referred to a basic coordinates $(\bar{x}, \bar{s}, \bar{r})$, in which $\bar{x}$ is the axis of shell length coordinate, $\bar{s}$ is the axis of arc length of the shell's cross section along the circumferential coordinate, $\bar{r}$ is the axis of radial coordinate. The axial, circumferential and radial displacements of the shell are represented by u, v, w, respectively. r_0 is the radius of a circular cross section with the same perimeter of elliptic cylindrical shell, recorded as the average radius of elliptic cylindrical shell. R is the curvature radius of the cross section, and is related to the coordinate $\bar{s}$, recorded as $R = R(\bar{s})$.

Dimensionless coordinates are introduced as follows:

$$x = \frac{\bar{x}}{r_0},\ s = \frac{\bar{s}}{r_0},\ r = \frac{R(\bar{s})}{r_0} = r(s) \tag{1}$$

2.2. Basic relations between the shell's displacements and internal forces

According to Flügge (1973) thin shell theory, the relations between strains and deformations of the shell are given as:

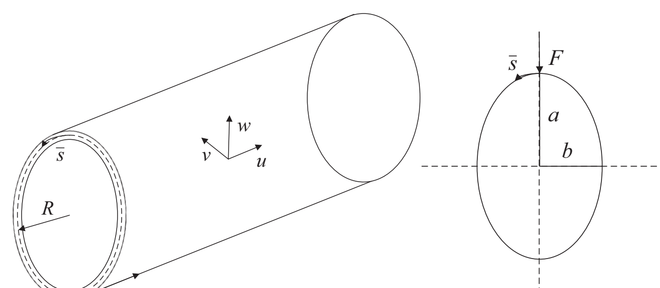

Fig. 1. Geometry parameters and coordinate system of elliptic cylindrical shell.