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On the stability of linear fractional-space neutron point kinetics (F-SNPK) models for nuclear reactor dynamics

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ABSTRACT

The aim of this work is to investigate the stability of linear fractional-space neutron point kinetics (F-SNPK) models for nuclear reactor dynamics, using three methods: root locus, Bode plot and unit step response. The F-SNPK is based on an approximation non-Fickian that is modeled considering that the differential operator of neutron density current is of fractional order, known as anomalous diffusion exponent. Two rector geometries for different values of anomalous diffusion exponent where analyzed. The results obtained with F-SNPK were compared with the classical neutron point kinetic (CNPK) equations. It has been found that the F-SNPK models are open-loop as well as closed-loop stable for both the geometries. The models are also found to exhibit faster dynamics with increase in subdiffusivity.

1. Introduction

The fractional-space neutron point kinetic (FNPK) approximation developed by Espinosa-Paredes (2017) is the fractional order (FO), which for one-group of neutron delayed precursors, is given by:

$$\frac{dn(t)}{dt} = \left[\frac{(\rho - \beta)}{\Lambda} + \underbrace{f(\alpha)}_{\text{FO term}}\right] n(t) + \lambda c(t), \quad \textit{for } 0 < \alpha < 1 \tag{1}$$

where the precursor concentration is given by:

$$\frac{dc(t)}{dt} = \frac{\beta}{\Lambda} n(t) - \lambda c(t) \tag{2}$$

The initial conditions are given by:

$$n(0) = n_0 \tag{3}$$

$$c(0) = \frac{\beta}{\lambda\Lambda} n_0 \tag{4}$$

with

$$\rho(0) = -\Lambda f(\alpha) \tag{5}$$

In these equations $n(t)$ is the neutron density; ρ is the reactivity; β is the total fraction of delayed neutrons; Λ is one generation average lifetime of instantaneous neutrons; λ is the decay constant of the delayed neutron precursor; $c(t)$ is the delayed neutron.

Eq. (1) has a new term respect to classical neutron point kinetics (CNPK), which represent the anomalous diffusion source (Espinosa-Paredes, 2017):

$$f(\alpha) = \upsilon(D_\gamma B_g^2 - D_{\alpha\gamma} B_g^{\alpha+1}), \tag{6}$$

where υ is the neutron speed, D_γ is the neutron diffusion coefficient, $D_{\alpha\gamma}$ is the fractional diffusion coefficient whose units are cm^α, B_g is the geometric buckling, α is the order of the differential operator known as the anomalous diffusion coefficient: for sub-diffusion process: $0 < \alpha < 1$; while for the super-diffusion process: $1 < \alpha < 2$. The concept of fractional divergence has also been discussed in (Das and Biswas, 2007).

Recently, there have been attempts (see, Espinosa-Paredes et al., 2013; Vyawahare and Nataraj, 2013a,b; Espinosa-Paredes, 2017) to model neutron transport in a nuclear reactor as anomalous diffusion, particularly, sub-diffusion (slow diffusion). The movement of neutrons inside a nuclear reactor core is not a simple diffusion process. The neutrons undergoing movements are captured for fission reactions, neutron capture, etc. These processes which capture neutrons can be treated as local traps with non-zero waiting times and hence the neutron transport can be modeled as sub-diffusion.

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The sub-diffusion of neutrons can be efficiently modeled using fractional-order neutron telegraph equations involving the time and or space derivatives of non-integer order. Consequently, the higher dynamic models of nuclear reactor which are based on the fundamental neutron movement equations are also of fractional order (for example, the fractional-order neutron point kinetic model). Thus, the sub-diffusive nature of neutron transport appears in terms of fractional-order differential equations in all type of reactors (viz. BWR, PWR, PHWR, etc). Also see (Moreno, 2010, Henrice Jr., et al., 2017). The solution of coupled FO diffusion equations has been presented in (Sardar et al., 2010). The numerical solution of neutron FO diffusion equation has been discussed in previous works (Moghaddam et al., 2014, 2015a–c).

The *anomalous diffusion source* contains two terms: the first term is of integer order (*IO term*), and the second term is the fractional order (FO term), as can be seen in this equation. The IO term is due to normal diffusion whose approximation is $\nabla^2\phi_\gamma(\mathbf{r},t) = -B_g^2\phi_\gamma(\mathbf{r},t)$ (Glasstone and Sesonske, 1981), while the FO term is due to anomalous diffusion, i.e. $\nabla^{\alpha+1}\phi_\gamma(\mathbf{r},t) = -B_g^{\alpha+1}\phi_\gamma(\mathbf{r},t)$.

The anomalous diffusion process can be introduced in normal neutron diffusion theory considering Non-Fickian approximation with a fractional constitutive equation of the current density vector (Espinosa-Paredes et al., 2013):

$$\mathbf{J}_\gamma(\mathbf{r},t) = -D_{\alpha\gamma}(\mathbf{r})\nabla^\alpha\phi_\gamma(\mathbf{r},t), \quad \textit{for } 0 < \alpha < 1 \tag{7}$$

where ϕ_γ is the neutron flux en el medium γ (Fig. 1), ∇^α is the differential operator of fractional order (FO), which involves fractional-order derivatives (${}_0D^\alpha$). The divergence of the current vector (Eq. (6)) is:

$$\nabla\cdot\mathbf{J}_\gamma(\mathbf{r},t) = -D_{\alpha\gamma}(\mathbf{r})\nabla^{\alpha+1}\phi_\gamma(\mathbf{r},t), \quad \textit{for } 0 < \alpha < 1 \tag{8}$$

The fractional-order derivative ${}_0D^\alpha$ can be defined in different ways from fractional calculus deals with the derivatives and integrals of non-integer, real or complex-order. The field is as old as the conventional calculus, originated in 1695. Riemann-Liouville defined the fractional differential as follows (Das, 2011):

$$D^\alpha\phi(x) = \frac{d^m}{dx^m}\left[\frac{1}{\Gamma(m-\kappa)}\int_0^x \frac{\phi(\tau)}{(x-\tau)^{\alpha+1-m}}d\tau\right], \quad \textit{for } (m-1) \leqslant \alpha < m \tag{9}$$

Caputo formulated the fractional differentials as follows:

$$D^\alpha\phi(x) = \frac{1}{\Gamma(m-\alpha)}\int_0^x \frac{\phi^{(m)}(\tau)}{(x-\tau)^{\alpha+1-m}}d\tau, \quad \textit{for } (m-1) \leqslant \alpha < m \tag{10}$$

Grünwald–Letnikov formulated the fractional derivatives as

$${}_aD^\alpha\phi(x) = \lim_{h\to 0}\frac{1}{h^\alpha}\sum_{m=0}^{\left[\frac{x-a}{h}\right]}(-1)^m\frac{\Gamma(\alpha+1)}{m!\Gamma(\alpha-m+1)}\phi(x-mh), \quad \textit{for } (m-1) \leqslant \alpha < m \tag{11}$$

In these equations m is the integer and α is a positive real number. Although the formation of Eqs. (9) to (11) are different; Podlubny (1999) proved that they are equivalent.

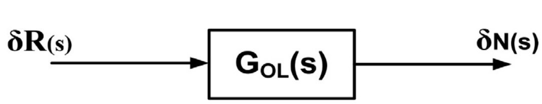

Fig. 1. Open-loop configuration.

The quantity $D_{\alpha\gamma}B_g^{\alpha+1}$ has the dimensions of $(length)^{-1}$, since $D_{\alpha\gamma}$ is a $(length)^\alpha$ and $B_g^{\alpha+1}$ is a $(length)^{-(\alpha+1)}$. Then, the quantities $D_\gamma B_g^2$ and $D_{\alpha\gamma}B_g^{\alpha+1}$, are consistent in dimensions. The geometric buckling of fractional order ($B_g^{\alpha+1}$) can be obtained of the solutions of normal diffusion (*integer order*), for different reactor geometries.

Now, when $\alpha \to 1$ the F-SNPK given by Eq. (1), lead to the classical neutron point kinetics (CNPK) equations.

$$\frac{dn(t)}{dt} = \frac{\rho-\beta}{\Lambda}n(t) + \lambda c(t). \tag{12}$$

In this work we investigate the stability of F-SNPK equations for nuclear reactor dynamics, using frequency domain method given by Eq. (1). Two rector geometries for different values of fractional-order derivative where analyzed. The results obtained with F-SNPK (Eq. (1)) are compared with the CNPK (Eq. (12)). The salient contributions of this work can be listed as follows:

1. Development of novel linear F-SNPK model for 1-group delayed neutrons.
2. Stability analysis of open- and closed-loop linear F-SNPK models using root locus, Bode plot and unit step response methods (for slab and cylindrical geometries).
3. Detailed analysis of the time and frequency domain performance indices of F-SNPK models and comparison with CNPK model.
4. Study of effect of subdiffusion parameter α on the dynamics of the F-SNPK models.

The paper is organized as follows. Next section gives the derivation of linear F-SNPK model. Section 3 briefly explains the stability analysis tools employed in this work and tabulates the parameter values considered. Stability analysis of open-loop CNPK and F-SNPK models is given in Section 4. Closed-loop stability analysis is discussed in detail in Section 5. Section 6 presents the conclusion.

2. Linear F-SNPK model

This section presents the development of linearized version of F-SNPK model. The well-known method of small-signal analysis is used to derive the linear model (Ogata, 1979; Khalil, 2002). The linear model is particularly useful in understanding the behavior of nuclear reactor for small changes in the input. The variation in neutron concentration when the reactor is excited by small changes in reactivity near the equilibrium condition is necessary to understand the reactor dynamics. The developed linear model is used to investigate the stability of the reactor.

The F-SNPK model is given by Eqs. (1) and (2), which are nonlinear model. In order to linearize the model, the model is expressed in terms of perturbation variables as follows. Assuming Caputo fractional derivatives,

$$\delta n = n - n^* \tag{13}$$

$$\delta c = c - c^* \tag{14}$$

$$\delta\rho = \rho - \rho^* \tag{15}$$

where n^*, c^* and ρ^* represent the values of these variables at the equilibrium condition.

Substituting in (1) and (2) gives

$$\frac{d}{dt}(\delta n + n^*) = \left[\frac{\delta\rho + \rho^* - \beta}{\Lambda} + f(\alpha)\right](\delta n + n^*) + \lambda(\delta c + c^*) \tag{16}$$

Neglecting small second order and higher order terms gives linear approximation of the above differential equation. The resulting linear differential equation in deviation variables is: