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Practical environment-corrected discontinuity factors and homogenized parameters for improved PT-SCWR neutron diffusion solutions

J.R. Sharpe *, A. Buijs

McMaster University, Hamilton, Ontario, Canada

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ABSTRACT

A novel application of practical discontinuity factors in coarse-mesh finite-difference solutions of heterogeneous multi cells with Pressure Tube Supercritical Water-cooled Reactor (PT-SCWR) type fuel and moderator cells has been investigated. In addition to discontinuity factors, homogenization schemes have also been studied and applied to reflector-adjacent-fuel cells and reflector cells. A 49 node inner-core multi cell model containing only fuel and a 40 node outer-core multi cell model containing reflector and fuel cells were simulated. In addition to nominal conditions, various discontinuity factors and homogenization techniques, along with conventional techniques, were applied to static beyond-nominal condition scenarios to evaluate the error associated with neutron power changes from nominal conditions, relative to reference transport solutions. The use of novel mean reference discontinuity factors and environment homogenized reflector-adjacent-fuel cells, over conventional methods, reduces core-wide reactivity errors, RMS neutron power errors, and maximum channel specific power errors by up to 2.6 mk, 2.9%, and 6.7%, respectively.

1. Introduction and background

Modern neutron transport codes are capable of accurately simulating neutronic interactions in a critical assembly. Unfortunately, the computational resources required to produce exact solutions restrict their use in routine calculations. For this reason, many approximate techniques have been developed that significantly reduce computation time at the cost of accuracy or generality. The strategy that has dominated rapid algorithms is the use of Fick's Law to convert the time-independent integro-differential neutron transport equation into a second-order heterogeneous differential equation: the neutron diffusion equation (Stacey, 2007). This strategy was met with mixed success due to assumptions made by Fick's law that do not translate well to every situation.

With Natural Resources Canada's development of a new nuclear power reactor, the Pressure Tube Supercritical Water-cooled Reactor (PT-SCWR), in collaboration with Canadian Nuclear Laboratories (CNL) and Canadian universities, the applicability of conventional nuclear simulation codes to the PT-SCWR must be determined. Approximations such as (1) assembly discontinuity factors (ADF) which are used in Pressurized Water Reactor (PWR) analysis, or (2) the non-use of discontinuity factors (equivalent to unitary discontinuity factors - UDF) in pressurized heavy-water reactors (PHWR), may not apply to the PT-SCWR. Thus, a novel approximate method has been developed and is presented in this paper.

1.1. The Canadian Pressure Tube Supercritical Water-cooled Reactor (PT-SCWR)

The PT-SCWR (Boyle et al., 2009) draws upon Canadian expertise through the use of pressure tubes similar to those used in a CANDU™ reactor. The pressure tubes proposed for the PT-SCWR are high-efficiency re-entrant flow channels (Pencer et al., 2012) and are vertically oriented to increase passive safety associated with thermal siphoning. A recent iteration of the PT-SCWR design, used in this work, was designed to operate at 2540 MWth with an electrical output of 1200 MW_e for a thermal efficiency $\eta_{th} = 48\%$.

The PT-SCWR lattice cell shown in Fig. 1 consists of an inner flow channel, two concentric rings of fuel pins with an outer flow channel bounded by insulators and a pressure tube, all of which are surrounded by a low pressure heavy-water moderator. The inner and outer rings of fuel pins are 15 wt% and 12 wt% reactor grade PuO_2 in ThO_2, respectively, with 32 fuel pins per ring. Supercritical light water coolant in the central flow tube descends to the bottom of the 5 m long channel and is subsequently redirected upwards to

* Corresponding author.
E-mail addresses: sharpejr@mcmaster.ca (J.R. Sharpe), buijsa@mcmaster.ca (A. Buijs).

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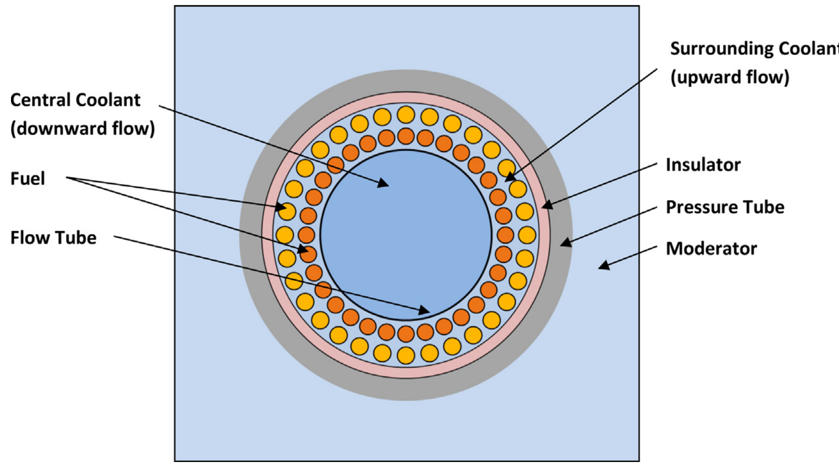

Fig. 1. Cross-sectional view of the PT-SCWR fuel cell.

pass by fuel pins to remove their generated heat. A side view of the PT-SCWR which is comprised of 336 vertical high efficiency re-entrant flow channels with an inlet and outlet plenum both located above the core is shown in Fig. 2.

1.2. Diffusion equation

Development of the PT-SCWR fuel cycle has been performed to date through the use of conventional diffusion codes developed in Canada and the USA. The conventional and modern approaches to solving the diffusion approximation are explained below.

Diffusion methods are based on a second-order heterogeneous differential neutron diffusion equation; however, further approximations can be made. One common approximation is the removal of the spatial continuity of the flux which transforms the diffusion equation into a coarse-mesh finite difference problem. Similarly, the energy dependency is discretized, in this case into two energy groups:

$$J_1 + (\Sigma_{tt1} + \Sigma_{s12} - \nu\Sigma_{f1}/k)h_x h_y \phi_1 + (-\Sigma_{s21} - \nu\Sigma_{f2}/k)h_x h_y \phi_2 = 0, \tag{1}$$

$$J_2 + (-\Sigma_{s12})h_x h_y \phi_1 + (\Sigma_{tt2} + \Sigma_{s21})h_x h_y \phi_2 = 0, \tag{2}$$

where $J = (J_x^+ + J_x^-)h_y + (J_y^+ + J_y^-)h_x$ is the group-wise leakage out of a node[1] and $k = k_{\text{eff}}$ is the effective multiplication constant. The x and y dimensions of each node are h_x and h_y, respectively. In this work, only the x and y directions are considered, however, all calculations can trivially be extended to z. Σ_{tt} is the total-transfer cross section[2] which is the absorption cross section adjusted by the (n, xn) reactions; Σ_f is the fission cross section; ν is the average number of neutrons emitted per fission; and, Σ_{s12} and Σ_{s21} are the down-scatter and up-scatter cross sections, respectively.

In order to generate the cross sections listed in Eqs. (1) and (2), a neutron transport code must be applied to a given homogenization region. The fine-energy-group cross sections are subsequently collapsed in energy, and homogenized in space, into few-group cross sections to be used in Eqs. (1) and (2).

[1] In this convention a positive J is leakage out of a cell, and negative J is leakage into a cell.

[2] Also known as the total-scatter cross section.

1.3. Spatial homogenization and energy collapse

The spatial homogenization and energy collapse problems are conceptually simple: determine homogenized and collapsed cross sections such that a region's reaction rates and interface-averaged currents are identical to those of the heterogeneous solution provided by the transport code. A 1-D representation has been used to reduce the visual complexity of equations, however, they can trivially be extended to more dimensions. If nodally-averaged heterogeneous and homogeneous reactions rates are forced to be identical, then (Smith, 1986):

$$\int_{V_i} \hat{\Sigma}_{x,g}(r)\hat{\phi}_g(r)d^3r = \int_{V_i} \Sigma_{x,g}(r)\phi_g(r)d^3r. \tag{3}$$

Likewise, if the interface-averaged currents are forced to be identical, then:

$$\oint_{S_i^k} \widehat{\boldsymbol{J}}_{\boldsymbol{g}}(\boldsymbol{r}) \cdot dbiS = \oint_{S_i^k} \boldsymbol{J}_{\boldsymbol{g}}(\boldsymbol{r}) \cdot d\boldsymbol{S}, \tag{4}$$

where the hat ($\hat{\ }$) indicates a homogenized parameter, the bold font indicates a vector, S_i^k is the kth surface of homogenized region i, and x represents a particular reaction (not the spatial variable). Since the homogenized parameters are assumed to be constant over a homogenized region, the homogenized cross sections are:

$$\hat{\Sigma}_{x,g}(r) = \frac{\int_{V_i} \Sigma_{x,g}(r)\phi_g(r)d^3r}{\int_{V_i} \hat{\phi}_g(r)d^3r}. \tag{5}$$

If Fick's law (Stacey, 2007) ($J = -D\nabla\phi(r)$) is applied to the RHS of Eq. (4), then:

$$\widehat{D}_{i,g}^k = \frac{-\oint_{S_i^k} \boldsymbol{J}_{\boldsymbol{g}}(\boldsymbol{r}) \cdot d\boldsymbol{S}}{\oint_{S_i^k} \nabla\hat{\phi}_g(r) \cdot d\boldsymbol{S}} \tag{6}$$

In the coarse-mesh finite difference (CMFD) approximation, cross sections in a homogenization region (also known as a node) are assumed to be constant, along with the few-group fluxes. The difficulty to exactly satisfy Eqs. (5) and (6) is that the solution to the heterogeneous problem must be known in advance, in addition to the solution of the homogeneous problem. Specifically, the $\hat{\phi}_g(r)$ solution relies on nodal coupling, which depends on the homogenized $\widehat{D}_{i,g}^k$; however, $\widehat{D}_{i,g}^k$ is also generated from the $\nabla\hat{\phi}_g(r)$ in the denominator of Eq. (6). In practice, various approximation techniques must be used to address the issue of reaction rate and interface-averaged current equivalence between homogenized