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Review

A modified power method for the multilayer multigroup two-dimensional neutron diffusion equation

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ABSTRACT

In this work we present a methodology of solution of the multigroup multi-layer stationary neutron diffusion equation in two-dimensional cartesian geometry. This eigenvalue problem describes the criticality of nuclear reactor, that is, it establishes the ratio between the numbers of neutrons generated in successive fission reactions. In order to solve this problem, we use the power method to obtain the dominant eigenvalue (K_{eff}) and its corresponding eigenfunction. Each iteration of the power method requires the solution of a non–homogeneous diffusion problem, that usually is solved numerically, however in this work the neutron diffusion equation is solved in analytical form in each iteration. To solve this system of second order partial differential equations, we propose to use the Finite Fourier Transform in one of the spatial variables obtaining a transformed problem which is resolved by well-established methods for ordinary differential equations. After it is solved, we use the inverse Fourier Transform to reconstruct the expression of the neutron flux in the original variables. However, at each iteration of the power method it is necessary to update the source term with the neutron flux and the K_{eff} of the previous iteration. Thus in all iterations new terms are added which becomes the process very laborious. To overcome this problem, the authors propose a methodology that approximates the neutron flux in standard form by polynomial interpolation. In order to reduce computational time we propose to subdivide the real regions of the problem into small fictitious regions. In this way, the interpolating polynomials of each region can be of low order, reducing the dimensions of the matrices involved and, consequently, computational time. The methodology is implemented to solve a heterogeneous problem and the numerical results are compared with the finite volumes method.

Contents

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1. Introduction

Over the years, many methods to predict the neutron flux in a nuclear reactor have been developed. The most precise way to get the neutron flux is solving the neutron transport equation. However this equation is usually difficult to solve and it takes too much (computational) time to achieve results even with the current computation speeds. Therefore, certain approximations based on Fick's law are made to obtain the neutron flux, one of the classic approximations is the neutron diffusion equation (Sekimoto, 2007). This equation is used because it produces good results and has low computational cost compared to transport equation.

The steady state neutron diffusion equation is an eigenvalue problem, where only the dominant eigenvalue and its respective eigenfunction have physical meaning for the problems of interest. These calculations are important because they represent the neutron flux distribution for most of time dependent problems. In other words, the usual initial conditions for these initial value problems are the result from the neutron diffusion eigenvalue problem. The dominant eigenvalue is called the effective multiplication factor (K_{eff}), and it represents the ratio between neutron production (by fission) and loss from one generation to another. Also, the power method is the most used method to obtain the dominant eigenvalue and its eigenfunction (Duderstadt and Hamilton, 1976; Sekimoto, 2007).

Usually, the neutron diffusion equation is solved by numerical methods. Frequently these methods are based on dividing the domain into a mesh of points or nodes (spacial discretization technique) (Maiani and Montagnini, 2004; Rokrok et al., 2012; Bernal et al., 2014; Welch et al., 2017).

The analytical methods are less common in the literature, but not least important, since they provide extremely important benchmarks problems for the licensing and operation of the nuclear power plants. Usually an integral transform is applied to the set of equation, for example, Laplace Transform Technique (Lemos et al., 2008). More recently, a power series expansion was used (Ceolin et al., 2015).

In this article, we present the Fictitious Borders Power Method (FBPM). FBPM works for multigroup multi-layer stationary neutron diffusion equation in two-dimensional cartesian geometry. This method consists in dividing the domain into even smaller regions (fictitious regions). The neutron diffusion equation is solved via the power method for each fictitious region. However, for each iteration of the power method these equations are solved analytically using Fourier Transform method. The neutron fluxes are reconstructed by polynomial interpolation in each iteration to calculate the fictitious source term for the power method. Where the main advantages of this methodology are an analytical solution for each fictitious regions and a low time scale to obtain the numerical results.

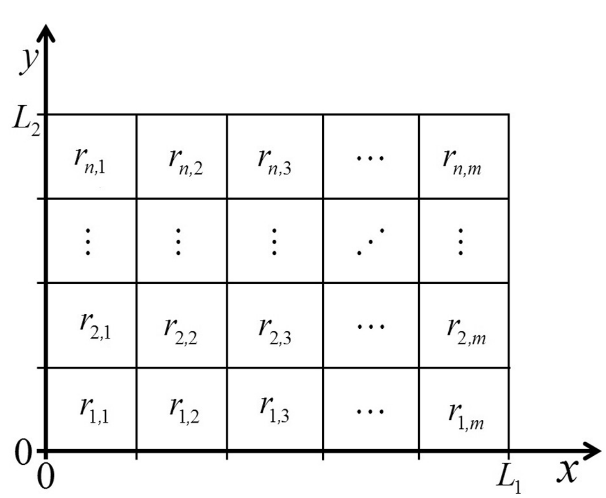

Fig. 1. Example of a multilayer two-dimensional problem in cartesian geometry.

2. Mathematical formulation

The multilayer multigroup two-dimensional neutron diffusion equation without external source and homogeneous nuclear parameters by regions writes:

$$-D_g^{(r)}\nabla^2\phi_g^{(r)} + \Sigma_{Rg}^{(r)}\phi_g^{(r)} = Q_g^{(r)}, \tag{1}$$

where $g = 1, \dots, G$ are neutron energy groups, $\phi_g^{(r)} = \phi_g^{(r)}(x,y), 0 \leqslant x \leqslant L_1, 0 \leqslant y \leqslant L_2$ and $Q_g^{(r)}$ is source term that contains the fission and scattering terms given by:

$$Q_g^{(r)} = \frac{1}{K_{eff}}\chi_g\sum_{g'=1}^{G}\nu_{g'}\Sigma_{fg'}^{(r)}\phi_{g'}^{(r)} + \sum_{\substack{g'=1\\ g'\neq g}}^{G}\Sigma_{sg'g}^{(r)}\phi_{g'}^{(r)}. \tag{2}$$

The notations in Eqs. (1) and (2) are: r are the regions of problem, according to Fig. 1; $D_g^{(r)}$ is the diffusion coefficient of the energy group g in region r; $\phi_g^{(r)}(x,y)$ is the neutron scalar flux of the energy group g in region r; $\Sigma_{Rg}^{(r)}$ is the removal cross section of the energy group g in region r; K_{eff} is the effective multiplication factor; χ_g is the integrated fission spectrum of the energy group g; ν_g is the average number of neutrons emitted by fission of the energy group g; $\Sigma_{fg}^{(r)}$ is the fission cross section of the energy group g in region r; $\Sigma_{sg'g}^{(r)}$ is the scattering cross section from energy group g' to g in region r.

To simplify the manipulations, it is convenient to rewrite the eigenvalue problem (1) and (2) into a matrix system in the following form:

$$\mathbf{M}\Phi = \frac{1}{K_{eff}}\mathbf{F}\Phi, \tag{3}$$

where the vector of the group neutron flux is $\Phi = [\,\phi_1 \quad \phi_2 \quad \cdots \quad \phi_G\,]^T$, the coefficient matrix $\mathbf{M} = (a_{ij})_{G,G}$ is defined as:

$$a_{ij} = \begin{cases} -D_i\nabla^2 + \Sigma_{Ri}, & i = j \\ -\Sigma_{sj'i}, & i \neq j \end{cases} \tag{4}$$

and the coefficient matrix $\mathbf{F} = (b_{ij})_{G,G}$ is defined as:

$$b_{ij} = \chi_i\nu_j\Sigma_{fj}.$$

In this work we consider a rectangular geometry with boundary conditions:

$$\begin{aligned} &\alpha_{1g}\phi_g|_{\partial\Gamma_{x,0}} + \beta_{1g}\frac{\partial\phi_g}{\partial x}\bigg|_{\partial\Gamma_{x,0}} = 0; \\ &\alpha_{2g}\phi_g|_{\partial\Gamma_{0,y}} + \beta_{2g}\frac{\partial\phi_g}{\partial y}\bigg|_{\partial\Gamma_{0,y}} = 0; \\ &\alpha_{3g}\phi_g|_{\partial\Gamma_{x,L_2}} + \beta_{3g}\frac{\partial\phi_g}{\partial x}\bigg|_{\partial\Gamma_{x,L_2}} = 0; \\ &\alpha_{4g}\phi_g|_{\partial\Gamma_{L_1,y}} + \beta_{4g}\frac{\partial\phi_g}{\partial y}\bigg|_{\partial\Gamma_{L_1,y}} = 0, \end{aligned} \tag{5}$$

where α and β are constants and $|\alpha| + |\beta| > 0$. Also, the flux and current in x and y directions continuities are valid in the whole domain,