Annals of Nuclear Energy 111 (2018) 666–675

Contents lists available at ScienceDirect

Annals of Nuclear Energy

journal homepage: www.elsevier.com/locate/anucene

Estimation of reactivity and delayed neutron precursors' concentrations using a multiscale extended Kalman filter

Shrenik B. Patel [a,*], S. Mukhopadhyay [a,b], A.P. Tiwari [a,b]

[a] Homi Bhabha National Institute, Trombay, Mumbai 400094, India
[b] Bhabha Atomic Research Centre, Trombay, Mumbai 400085, India

ARTICLE INFO

Article history:
Received 15 March 2017
Received in revised form 24 August 2017
Accepted 17 September 2017
Available online 7 October 2017

Keywords:
Nuclear reactor
Wavelet filters
Extended Kalman filter
Reactivity meter

ABSTRACT

A wavelet based multiscale extended Kalman filtering technique for estimation of reactivity and delayed neutron precursors' concentrations is presented in this paper. Reactivity which indicates the criticality status of the reactor core can only be measured in indirect way. Similarly delayed neutron precursors' concentrations, the source of the delayed neutrons which play important role in reactor control cannot be measured directly. Nuclear reactor is an example of multirate nonlinear system in which different state variables evolve with widely varying dynamics. The state estimation algorithm presented here is based on and preserves merits of Extended Kalman Filtering (EKF) technique. In addition, use of wavelet filters enables multiscale decomposition of the state variables that in turn, effectively captures the multirate nature of the system. Estimation has been carried out using reactor power as the only input. In order to justify effectiveness of the proposed method, simulation results are shown for completely known power variation dataset and experimental power variation datasets collected from one of the Indian research reactors.

1. Introduction

Reactivity in the nuclear reactor is a very important variable which indicates the status of the reactor core. Online measurement of reactivity is necessary for calibration of control devices, monitoring shutdown margins, quantification of the worth of fuel bundles, etc. Shimazu et al. (1987) and Ma et al. (2012). Other important variables are concentrations of the delayed neutron precursors which emit delayed neutrons that in turn, play important role in reactor control (Duderstadt et al., 1976).

Due to lack of suitable sensors, variables like reactivity and delayed neutron precursors' concentrations cannot be measured directly (Perez-Cruz et al., 2007). Consequently, these variables need to be estimated from neutron flux/power measurements using an appropriate estimation algorithm (Simon, 2006). Conventionally Inverse Point Kinetics (IPK) like algorithms were used for the purpose (Ansari, 1991). However, these techniques assume system variables to be deterministic. But in reality, measurement signals are inevitably corrupted with noise arising from detectors, communication channels and over and above due to fundamental stochastic nature of the fission process itself. The classical techniques do not address stochastic behaviour of reactor kinetics and measurement process. Commonly used method to eliminate noise is to employ a low pass filter but it may as well remove some of the information rich frequency components generated by the reactor system.

Kalman filter is one of the promising modern optimal state estimation algorithms that permits working in a stochastic framework under the assumption that process uncertainties and measurement noise have Gaussian distribution. Many attempts have been made for the observer design problem of the nuclear reactor using Kalman filter. Racz (1992) proposed Kalman filtering method for reactivity estimation for small changes in the reactivity. Dong (2010) reported application of robust Kalman filter to estimate various state variables of a reactor such as neutronic flux, concentration of delayed neuron precursors, average fuel temperature, coolant temperature inside the reactor and coolant temperature entering the reactor core. Bhatt et al. (2013) reported Extended Kalman Filtering (EKF) technique for online reactivity estimation of the nuclear reactor and justified merits of the EKF technique over conventional IPK like technique. Silva et al. (2015) demonstrated simulation results for reactivity estimation during reactivity initiated accidents using EKF technique. Shimazu and van Rooijen (2014) reported qualitative performance comparison between IPK and EKF techniques. Peng et al. (2016) demonstrated comparative study of two EKF techniques using different Jacobian structures. Ygane

* Corresponding author.
E-mail address: shrenik@barc.gov.in (S.B. Patel).

https://doi.org/10.1016/j.anucene.2017.09.033
0306-4549/

and Ansarifar (2017) reported EKF technique to estimate the poisons' concentrations in the PWR nuclear reactors based on the reactor power measurement. They have carried out a comparative study of the results obtained from the continuous time EKF with those from KF and Luenberger observer. An important feature of the Kalman filter algorithm is that almost all the available information about the system can be used to improve estimator's performance.

In past few decades wavelets and filter bank theory have emerged as an effective way for multiscale analysis of the signals and systems (Daubechies, 1992; Strang and Nguyen, 1996; Vetterli and Strang, 1994; Torrence and Compo, 1998; Crouse et al., 1998). A wavelet based Kalman filter for estimation of random signals from multiscale decomposition has been presented by Hong et al. (1998). They reported an interesting way for simultaneous decomposition and estimation for a class of autonomous systems. In this article we move one step further to design a multiscale EKF algorithm for a multivariable nonlinear system in general and use it for estimation of the state variables of a nuclear reactor. Motivation for this work stems from the fact that a nuclear reactor is a nonlinear multiscale system in which system states with widely varying dynamics evolve simultaneously. It is demonstrated that an underlying multiscale nonlinear model is likely to capture these modes of the evolution better than that by a conventional model in single resolution.

In this paper, a mutliscale model structure is formulated by projecting system states on Haar scaling and wavelet functions as basis and EKF algorithm is implemented on the system model in projection space. The proposed algorithm is tested and its performance is compared with the standard EKF algorithm for the data sets obtained from a research reactor. From the simulation results, it is found that if decomposition is performed up to the certain scale, the multiscale EKF algorithm outperforms the standard EKF algorithm.

The rest of the paper is organized as follows. Section II describes design methodology used for the estimation purpose. Section III describes simulation results for the dataset generated by known reactivity variation and experimental datasets collected from a research reactor. Conclusions are drawn in section IV to indicate major achievements of the work.

2. Design methodology

In this section, state estimation using EKF and extension of this technique for the multiscale model is presented. The point kinetics model (Hetrick, 1971) assuming small change in the reactivity is used for the purpose of estimation. It can be expressed as:

$$\dot{n} = \left(\frac{\rho-\beta}{l}\right)n + \sum_{i=1}^{6}\lambda_i C_i \tag{1}$$

$$\dot{C}_i = \frac{\beta_i}{l}n - \lambda_i C_i, \quad i = 1,2,\cdots,6 \tag{2}$$

where state variables ρ, n and C_i indicate reactivity, neutronic power and delayed neutron precursor concentration of the ith group respectively. β_i and λ_i are fraction and decay rate of the delayed neutron precursors of the ith group with $\beta = \sum_{i=1}^{6}\beta_i$. l denotes prompt neutron lifetime.

Reactivity in (1) is an input to the system which is generally unknown and it is approximated by a random walk model (Maybeck, 1982), i.e.,

$$\dot{\rho} = 0. \tag{3}$$

2.1. Estimation using EKF

The Kalman Filter is a predictor–corrector type optimal state estimator which works with the assumption that process uncertainties and measurement noise have Gaussian distribution. The algorithm consists of two steps. In the first step, current estimate of the state variables and error covariance are forwarded into next time instance. In the next step, new measurement information is fused with the current estimate such that the covariance of error becomes minimum.

In order to employ EKF, the reference model given by (1) needs to be transformed into the state space form as:

$$\dot{x} = f(x) = F_n x \tag{4}$$

where the matrix F_n and the state vector x are defined as:

$$F_n = \begin{bmatrix} \frac{-\beta}{l} & \lambda_1 & \lambda_2 & \cdots & \lambda_6 & \frac{n}{l} \\ \frac{\beta_1}{l} & -\lambda_1 & 0 & \cdots & 0 & 0 \\ \frac{\beta_2}{l} & 0 & -\lambda_2 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \frac{\beta_6}{l} & 0 & 0 & \cdots & -\lambda_6 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \tag{5}$$

$$x = [\,n \quad C_1 \quad C_2 \quad C_3 \quad C_4 \quad C_5 \quad C_6 \quad \rho\,]^\top \tag{6}$$

The superscript $\top$ denotes transpose of a vector. System represented by (4) is nonlinear due to the presence of the state variable n in F_n as well as in state vector x. Jacobian of this nonlinear system can be defined as:

$$F = \frac{\partial f(x)}{\partial x} \tag{7}$$

All the elements of matrix F will be same as the corresponding elements of the matrix F_n except the element $F(1,7)$ which is given as $F(1,7) = \frac{n}{l}$.

In order to apply EKF in discrete domain, (4) must be transformed into a set of difference equations. If sampling is carried out at uniform interval of T_s seconds, set of difference equations can be written as

$$\begin{aligned} x_k &= \Phi_{n,k-1}x_{k-1} + w_{k-1} \\ \Phi_{n,k} &= e^{F_{N,k}T_s} \end{aligned} \tag{8}$$

where the subscript k denotes the sampling instant. w is the system uncertainty which is assumed to have zero mean and covariance Q. $F_{n,k}$ denotes the system matrix F_n defined by (5) at kth sampling instant. Similarly, discretization of Jacobian matrix is represented by $\Phi_k = e^{F_k T_s}$ where F_k denotes the Jacobian matrix F defined by (7) at the kth sampling instant. The measurement process is governed by the following equation:

$$z_k = Hx_k + v_k \tag{9}$$

where v is measurement noise with zero mean and covariance R and

$$H = [\,1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0\,]. \tag{10}$$

Equations for two step prediction-correction algorithm of EKF can be written as follows Sorenson (1985):

State Prediction:

$$\hat{x}_k^- = \Phi_{n,k-1}\hat{x}_{k-1}^+ \tag{11}$$

$$P_k^- = \Phi_k P_{k-1}^+ \Phi_k^\top + Q_k \tag{12}$$

where $\hat{x}_k^-$ is *a priori* estimate of the state at instant k given the knowledge of the process prior to instant k, $\hat{x}_{k-1}^+$ is *a posteriori* estimate of the state at instant $k-1$. $P_k^- = E\left[(\hat{x}_k^- - x_k)(\hat{x}_k^- - x_k)^\top\right]$ is *a priori* estimate of the error covariance at instant k and