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The use of representativity theory in the depletion calculations of SFR blankets

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ABSTRACT

The analysis of the DOUBLON experiment has provided C/E's for several isotopic ratios in the first and second rows of the radial blanket after an irradiation in the PHENIX Sodium Fast Reactor. We have used this as a basis for the validation of the DARWIN-2.3 (Tsilanizara et al., 2000) package for the depletion calculation in the radial blankets of SFR's (Lebrat et al., 2015). Unfortunately, no measurements have been performed in the upper and lower axial blankets of the fissile assemblies. Of course we expect our calculations to be reliable in these zones as well, as the initial compositions, nuclear reactions and neutron spectra are very similar. The representativity theory appeared to be the appropriate tool to correlate the available experimental information in the radial blanket to our calculations in the axial zones. Indeed, the representativity factors that we calculate between the final $^{239}Pu/^{238}U$ ratio in the radial and axial fertile zones are very close to 1. Hence, the measurements performed in the radial zone can help us "correct" the calculation in the axial blanket and reduce its a-priori uncertainty due to nuclear data. The estimated uncertainty reduction can reach a factor 5, since the experimental uncertainty is low. The $^{240}Pu/^{238}U$ ratio is studied with a similar method.

1. Introduction

The analysis of the DOUBLON experiment has provided C/E's for several isotopic ratios in the first and second rows of the radial blanket after an irradiation in the PHENIX Sodium Fast Reactor. We have used this as a basis for the validation of the DARWIN-2.3 (Tsilanizara et al., 2000) package for the depletion calculation in the radial blankets of SFR's (Lebrat et al., 2015). Unfortunately, no measurements have been performed in the upper and lower axial blankets of the fissile assemblies. Of course we expect our calculations to be reliable in these zones as well, as the initial compositions, nuclear reactions and neutron spectra are very similar. The representativity theory appeared to be the appropriate tool to correlate the available experimental information in the radial blanket to our calculations in the axial zones.

2. Definition of the problem

The parameter we have chosen to study is the final $^{239}Pu/^{238}U$ concentrations ratio in the blanket after irradiation and cooling. Indeed, the ^{239}Pu is the main isotope produced in the fertile zones, whereas the amount of ^{238}U is almost constant. This makes the final $^{239}Pu/^{238}U$ ratio a good measurement of the "burnup" in the blankets.

The ^{239}Pu is produced in the fertile zones by a capture reaction on the initial ^{238}U (via the ^{239}Np), so the final amount of ^{239}Pu depends on the neutron spectrum (via the cross sections) and the decay constants of ^{239}Np and ^{239}Pu. These dependencies are quantified by relative sensitivity coefficients: they measure the response of an integral parameter to a variation of a nuclear data. They are calculated by using the perturbation theory (Kallfelz et al., 1997; Williams, 1978, 1986) and can be written in our case:

$$S_\sigma = \frac{\sigma}{N} \times \frac{\partial N}{\partial \sigma} \quad (1)$$

where S is a vector with $N_{isotopes} \times N_{reactions} \times N_{groupes}$ entries, to which we must add the decay constants of the radioactive isotopes involved.

The sensitivity coefficients are computed for each isotope, each reaction and each of the 33 energy groups by the MECCYCO module of the ERANOS package (Ruggieri et al., 2006). This program performs adjoint multigroup evolution calculations, in order to obtain adjoint concentrations (i.e. importances). The relative sensitivities on the $^{239}Pu/^{238}U$ ratio are then obtained by a simple relationship:

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$$S^{\sigma}_{\left(\frac{Pu239}{U238}\right)} = \frac{\sigma}{\left(\frac{Pu239}{U238}\right)} \times \frac{\partial}{\partial\sigma}\left(\frac{Pu239}{U238}\right)$$

$$S^{\sigma}_{\left(\frac{Pu239}{U238}\right)} = \frac{\sigma}{\left(\frac{Pu239}{U238}\right)} \times \left(\frac{1}{U238}\right)^2 \left[U238 \times \frac{\partial}{\partial\sigma}(Pu239) - Pu239 \times \frac{\partial}{\partial\sigma}(U238)\right]$$

$$S^{\sigma}_{\left(\frac{Pu239}{U238}\right)} = \frac{\sigma}{Pu239} \times \frac{\partial}{\partial\sigma}(Pu239) - \frac{\sigma}{U238} \times \frac{\partial}{\partial\sigma}(U238)$$

$$S^{\sigma}_{\left(\frac{Pu239}{U238}\right)} = S^{\sigma}_{Pu239} - S^{\sigma}_{U238} \tag{2}$$

where "^{239}Pu" and "^{238}U" are the final concentrations of each isotope. We observe that the relative sensitivity on the isotopic ratio is simply the difference of the individual relative sensitivities.

Once the relative sensitivity coefficients are calculated, the uncertainty ε (in %) on any integral (in term of energy) parameter is then given by:

$$\varepsilon = \sqrt{S^t D S} \tag{3}$$

where D is the variance-covariance matrix of the nuclear data (the cross entries are in $[\%]^2$).

The representativity coefficient (Orlov, 1980) is usually used to compare a reactor concept with an experiment (Dos Santos et al., 2013a). We use it here in order to give the "similarity", the ≪correlation≫ between the ^{239}Pu/^{238}U ratios in two different zones 1 and 2 of a reactor by using their respective sensitivity vectors to nuclear data S_1 and S_2:

$$r_{1,2} = \frac{S_1^t D S_2}{\sqrt{S_1^t D S_1}\sqrt{S_2^t D S_2}} \tag{4}$$

D is the covariance matrix of the cross sections involved; we use the COMAC-V0 release (Archier et al., 2014) in a 33 energy groups mesh, based on the JEFF-3.1.1 library (JEFF-3.1.1, 0000).

Physically, r represents how much information the two zones "have in common" for a given parameter.

- If the two sensitivity vectors are identical, then r is maximal $r = 1$,
- The more different the sensitivities are, the more r decreases,
- If $r = 0$, it means that the productions of ^{239}Pu in the two zones are not correlated at all.

In our case, the covariance matrix must take into account the capture (c) and fission (f) of ^{238}U, ^{239}Np and ^{239}Pu. The two last diagonal entries $D_{199,199}$ and $D_{200,200}$ account for the decay of ^{239}Np and ^{239}Pu and finally D can be written as:

$$D = \begin{pmatrix} \begin{pmatrix} U^8_{ff} & U^8_{fc} \\ U^8_{cf} & U^8_{cc} \end{pmatrix} & 0 & 0 & 0 \\ 0 & \begin{pmatrix} Pu^9_{ff} & Pu^9_{fc} \\ Pu^9_{cf} & Pu^9_{cc} \end{pmatrix} & 0 & 0 \\ 0 & 0 & \begin{pmatrix} Np^9_{ff} & Np^9_{fc} \\ Np^9_{cf} & Np^9_{cc} \end{pmatrix} & 0 \\ 0 & 0 & 0 & \begin{pmatrix} D_{199,199} & 0 \\ 0 & D_{200,200} \end{pmatrix} \end{pmatrix} \tag{5}$$

This matrix represents the uncertainties and correlations, which cover the different groups as well as the different reactions. For instance, U^8_{ff} is a 33 * 33 matrix, whose:

- Diagonal entries are the squares of the relative uncertainties (in %) of the ^{238}U fission cross section for each energy group,
- The non-diagonal entries are the correlations between the different energy groups for the ^{238}U fission reaction.

The other square blocks U^8_{fc}, U^8_{cf} and U^8_{cc} are defined the same way and similar matrixes are defined for the other isotopes. It must be emphasized that D contains correlations between the various reactions for a given isotope, but not yet between different isotopes.

The two last diagonal entries $D_{199,199}$ and $D_{200,200}$ are the squares of the relative uncertainties on the decay constants λ of ^{239}Np (β-decay) and ^{239}Pu (α decay). They are deduced from the half-lifes T available in JEFF-3.1.1 (0000) by the following formula:

$$\left(\frac{\delta\lambda}{\lambda}\right)^2 = \left(\frac{\delta T}{T}\right)^2 \tag{6}$$

The values of $D_{199,199}$ and $D_{200,200}$ are calculated in Table 1.

So finally D will have $(6 \times 33 + 2)^2 = 200^2$ entries.

3. Application to our study

For our application, the above parameters are calculated in the positions of the PHENIX reactor that are represented on Fig. 1:

- At the core midplane of the inner fuel assembly (FUEL) located in position 18/20,
- At mid-height of the lower (CAI) and upper (CAS) axial blankets of the same fuel assembly,
- In the first row of the radial blanket (FEF), at the core midplane of the assembly 24/13,
- In the second row of the radial blanket (FEG), at the core midplane of the assembly 24/12.

The 42.8 EFPD's of the cycle 10 of the PHENIX reactor are computed with the ERANOS package and its integrated depletion module MECCYCO, with the same calculation route that was used for the interpretation of the TRAPU and DOUBLON irradiations (Lebrat et al., 2015). The sensitivities are evaluated in "fresh" fertile blankets composed of natural (for the CAI) or depleted (for the CAS, FEF and FEG) Uranium.

Fig. 2 shows the 33 energy group neutron spectrum obtained in the different zones of the reactor that we have studied; it is very similar in the various fertile zones and much "harder" in the fuel.

The same comment can be made on the sensitivity profiles shown on Figs. 3–6; to make the comparison easier on these figures, each sensitivity coefficients is normalized by its integral over energy. By definition, the normalized relative sensitivity coefficients are all positive.

Once these sensitivities are calculated, we use MATLAB to perform the calculation of the representativity factors between two zones 1 and 2 as defined by Eq. (4). The results are detailed in Table 2, as well as the uncertainties due to nuclear data ε that are defined by Eq. (3).

Table 2 shows that the representativity factors between any two of the various fertile zones are very close to 1. Of course, this was very predictable because the sensitivity profiles are very close

Table 1
Variances of the ^{239}Np and ^{239}Pu decay constants.

	T(s)	δT(s)	$\left(\frac{\delta T}{T}\right)^2$ (%^{2})
^{239}Np (β-)	$2.0347 \cdot 10^5$	$3.456 \cdot 10^2$	$2.885 \cdot 10^{-2}$
^{239}Pu (α)	$7.6094 \cdot 10^{11}$	$3.4713 \cdot 10^8$	$2.081 \cdot 10^{-3}$