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The role of neutron importance in bilinear weighted cross sections for burnup evaluation

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ABSTRACT

This work presents the development and implementation of bilinear weighted cross-sections for improving the accuracy of isotopic concentration calculations, considering neutron importance function in burnup evaluation. Monte Carlo based MCNIC method is used for calculation of the required neutron importance function. A module is developed for this purpose and is implemented into the IRBURN code system. Role of the neutron importance function in accuracy and precision of the burnup calculation is evaluated. An OECD/NEA experimental benchmark problems is used for this evaluation. The results are indicative of the fact that using the bilinear weighted cross-sections give a better estimation of the isotopic concentration in the burnup calculation for most isotopes.

1. Introduction

The coupled Bateman and transport equations govern the time dependent behavior of reactor fuel burnup and its isotopic inventory. These equations are the subject of research and software development by various researchers considering the effects of neutronic and depletion parameters in the burnup calculation (Morimoto et al., 1989; Kelly, 1995; Trellue and Poston, 1999; Okumura et al., 2000; Jafarikia et al., 2010). All these burnup codes use the following procedure in the calculation. The solution schemes include a quasi-static approach, which calculates the neutron spectrum and spectrum averaged cross-sections over a small time step by a neutronic code (Bell and Glasstone, 1970; Henry, 1975). Then, material composition is calculated in a depletion code by assuming that the spectrum as well as the spectrum averaged cross-sections are invariant in that time step. In all burnup codes, neutron cross-sections are weighted by the neutron spectrum for considering the spectrum evolution. But, the neutron importance is not considered in weighting procedure.

The concept of the adjoint function which is identified as neutron importance in the nuclear reactor field, has been introduced by Wigner (1945) and Lewins (1965). The neutron importance is interpreted in different ways depending on the object of interest. It depends on direction, energy and location of an inserted neutron (Henry, 1975). Previously, the adjoint equations for the constant flux approximation has been formalized by Williams (1979) leading to many extensions and generalizations (Yang and Downar, 1988; Da Silva and Thom, 1988; Downar, 1992).

In early works, the energy collapsing of fine group cross-section data for light water reactor analysis was performed with linear flux weighting. Several authors have attempted to use adjoint weighting in cross-section collapsing and investigated the merits of bilinear energy collapsing (Pitterle and Maynard, 1965; Henry, 1967; Buslik, 1968; Wade and Bucher, 1977; Kiefhaber, 1983). One of the primary motivations for bilinear studies has been that the bilinear weighting approach preserves reactivity worth which is particularly important for kinetics analysis (Lee et al., 2000). The most of those researches were related to fast reactors for which transient analysis was a particular concern (Wade and Bucher, 1977). Kim and Henry (1989) presented the flux-adjoint weighted cross-section for PWR reactor transient analysis. They used the continues energy P_1 equations for developing the bilinear weighted cross-sections.

The composition of the fuel changes during burn-up which in turn affects reactor properties such as neutron spectrum, reactor multiplication factor, etc. They imply uncertainties in the neutron spectrum, and it has to be propagated during the depletion analysis (Díez et al., 2015). Nuclear data uncertainties affect both transport and depletion codes (Downar, 1992; van Geemert and Hoogenboom, 2001; Christie et al., 2013; Kotlyar et al., 2017). Different methods can be used to get uncertainty quantification in the depletion calculation, and of course, these methods can be used with deterministic and Monte-Carlo approaches (Frosio et al., 2016). For example, Stripling et al. (2013) developed a general

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framework for computing the adjoint variable to nuclear engineering problems governed by a set of differential algebraic equations.

Flux and neutron importance can be determined by solving the forward and the adjoint transport equations based on deterministic methods. However, these calculations are very complicated in complex geometries. By using the Monte Carlo method, these functions can be determined by modeling and simulation of the reactor for different operating cycles.

By considering the capabilities of the MCNP code in solving problems with complex geometries and its closeness to physical concepts, a new method, MCNIC, based on the physical concept of neutron importance was introduced for the calculation of neutron importance in the nuclear reactor field by Feghhi et al. (2007). The efficiency of this method is benchmarked by comparison of the results with deterministic method calculations in the reactor core (Feghhi et al., 2007, 2008).

In what follows, we proposed bilinear weighting to take benefit from the neutron importance function for improving the accuracy of important averaged neutron cross-sections used in burnup calculations to improve the accuracy of core nuclide composition during burnup cycles. The MCNIC method is used for the calculation of neutron importance as adjoint weighting in the cross-section collapsing. This paper will focus on the efficiency of the bilinear weighted cross-sections in burnup problem with the calculation of the isotopic concentrations for the OECD/NEA benchmark problem.

2. Material and methods (Theory)

As is well known, fission products and actinide inventories, neutron flux as well as power distribution are interrelated and they are time dependent parameters. Moreover, the cross-section data generation and the neutron flux calculation in the core is crucial for enhancing the accuracy of calculations in the field of reactor physics.

The energy collapse of fine group cross-section data into few group constants for light water reactor analysis is performed with simple linear flux weighting. Many researchers do the burnup calculation by linear weighting. Unlike the linear weighting method, the bilinear weighting employs both the forward and adjoint spectra for group collapsing.

Coupling between the nuclide composition and the flux, plays a more significant role in thermal systems than fast ones. Our proposed depletion calculation method benefits from coupling between the core composition and the neutron spectrum as well as the adjoint flux via updating the averaged neutron cross-sections during fuel cycles.

2.1. Adjoint-weighted burnup calculation

2.1.1. Transport equation (forward and adjoint)

The fundamental equation describing the transport of radiation through a matter is the Boltzmann transport equation. The most widely known form of the equation is the Boltzmann equation in integral-differential form which reads in the general time dependent form:

$$\frac{1}{v}\frac{\partial\varphi(r,E,\Omega,t)}{\partial t}+\Omega.\nabla\varphi(r,E,\Omega,t)+\sum_t(r,E)\varphi(e,E,\Omega,t) = \int\int\sum_t(r,E')C(r,E'\to E,\Omega' \to\Omega)\varphi(r,E',\Omega',t)dE'd\Omega'+S(r,E,\Omega,t) \quad (1)$$

In which, E: the neutron energy, Ω: the flight direction, t: the time, v: the neutron speed, φ(r,E,Ω,t) dE dΩ: the neutron flux at r at time t for neutrons energy between E and E+dE, and direction in the solid angle dΩ about Ω, $\Sigma_t(r,E)$: the total cross-section at r for neutrons with energy E, $C(r,E'\to E,\Omega'\to\Omega)$ dE dΩ: the expected number of neutrons leaving a collision with energy between E and E+dE, and direction in the solid angle dΩ about Ω given a neutron with energy E′ and direction Ω′ going into a collision, S(r,E,Ω,t) is an external neutron source, i.e. independent of the neutron flux. The right hand side of Eq. (1) is the density of neutrons emitted at r with energy E and direction Ω from any collision process or the source (Hoogenboom, 1977).

The forward Boltzmann equation represents the neutron population quantity in the matter; but the neutron population quality is neglected. Hence, the adjoint Boltzmann equation is implemented to consider the neutron population quality. The mathematical basis of the adjoint flux is presented in the integral-differential form of the adjoint Boltzmann equation as follows:

$$-\frac{1}{v}\frac{\partial\varphi^\dagger(r,E,\Omega,t)}{\partial t}-\Omega.\nabla\varphi^\dagger(r,E,\Omega,t)+\sum_t(r,E)\varphi^\dagger(r,E,\Omega,t) = \int\int\sum_t(r,E')C(r,E\to E',\Omega \to\Omega')\varphi^\dagger(r,E',\Omega',t)dE'd\Omega'+S^\dagger(r,E,\Omega,t) \quad (2)$$

where all the symbols follow the same meaning as in Eq. (1) except for $\varphi^\dagger$(r,E,Ω,t) dE dΩ: the adjoint neutron flux at r at time t for neutrons energy between E and E+dE, and direction in the solid angle dΩ about Ω, $S^\dagger$ (r,E,Ω,t) is an adjoint external neutron source. Eqs. (1) and (2) can be solved and simulated with deterministic and Monte-Carlo approaches, respectively (Bell and Glasstone, 1970; Hoogenboom, 1977). However, numerical calculations in deterministic methods are very complicated in complex geometries. But, the neutron transport parameters can be directly determined by modeling and simulation of the reactor using a Monte Carlo approach even in complex geometries.

2.1.2. Depletion formulation

Time evolution of nuclide concentrations undergoing serial or linear decay chain is governed by a set of first-order differential equation, called Bateman equations. For complex radioactive decay schemes the equations differ from linear chain in that a higher number of coefficients and equations appears leading to complication of the solution. At this point, it can be expressed as follows (Croff, 1983):

$$\frac{dX_i(t)}{dt}=\sum_{j=1}^{N}l_{ij}\lambda_jX_j+\varphi\sum_{k=1}^{N}f_{ik}\sigma_kX_k-(\lambda_i+\varphi\sigma_i+r_i)X_i+F_i \quad (3)$$

where X_i = atom density of nuclide i, N = number of nuclides, l_{ij} = fraction of radioactive disintegration by other nuclide j which leads to formation of nuclide i, λ_j = radioactive decay constant for nuclide j, φ = position and energy-averaged neutron flux, f_{ik} = fraction of neutron absorption by other nuclide k which leads to formation of nuclide i, σ_k = spectrum-averaged neutron absorption cross-section of nuclide k, r_i = continuous fractional removal rate of nuclide i from the system, F_i = continuous feed rate of nuclide i. For contained solid fuels, r_i and F_i are zero.

In this work, we introduced the importance-spectrum weighted depletion equation to incorporate the neutron importance function in burnup calculation. The general form is described as follows:

$$\frac{dX_i(t)}{dt}=\sum_{j=1}^{N}l_{ij}\lambda_jX_j+\varphi\sum_{k=1}^{N}f_{ik}\sigma_k^*X_k-(\lambda_i+\varphi\sigma_i^*+r_i)X_i+F_i \quad (4)$$