Annals of Nuclear Energy 110 (2017) 1010–1022

Contents lists available at ScienceDirect

Annals of Nuclear Energy

journal homepage: www.elsevier.com/locate/anucene

Spatial modes for the neutron diffusion equation and their computation

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ARTICLE INFO

Article history:
Received 10 January 2017
Received in revised form 4 August 2017
Accepted 11 August 2017

Keywords:
Spatial modes
Finite element
Neutron diffusion equation
Block Newton method
Generalized eigenvalue problem

ABSTRACT

Different spatial modes can be defined for the neutron diffusion equation such as the λ, α and γ-modes. These modes have been successfully used for the analysis of nuclear reactor characteristics. In this work, these modes are studied using a high order finite element method to discretize the equations and also different methods to solve the resulting algebraic eigenproblems, are compared. Particularly, Krylov subspace methods and block-Newton methods have been studied. The performance of these methods has been tested in several 3D benchmark problems: a homogeneous reactor and several configurations of NEACRP reactor.

1. Introduction

Different kind of spatial modes have been defined for the neutron transport equation forcing the criticality of the system under study by modifying the cross-sections in different ways (Bell and Glasstone, 1970; Henry, 1975; Ronen et al., 1976), obtaining different eigenvalue problems known as the λ-modes, the α-modes and γ-modes. In Ronen et al. (1976), Velarde et al. (1978) these different modes are discussed and compared for fast neutron plutonium systems.

The multigroup diffusion equation is generally used to study reactor cores. These spatial modes can be also defined for this approximation and used for different purposes. In this way, the dominant λ-modes can be efficiently computed (Verdú et al., 1994; Verdú et al., 1999), and they have been used to study and classify the neutronic oscillations in BWR reactors (March-Leuba and Blakeman, 1991; Verdú et al., 1998; Miró et al., 2000) and to develop modal methods to solve the time dependent neutron diffusion equation (Miró et al., 2002).

The α-modes problem is basic in the field of nuclear reactor physics (Bell and Glasstone, 1970). These modes are important to develop monitoring techniques for subcritical systems (Lewins, 2013; Kópházi and Lathouwers, 2012; Uyttenhove et al., 2014). Efficient methods have been developed for the computation of the dominant α-modes of a reactor core using neutron diffusion equation (Modak and Gupta, 2007; Verdu et al., 2010; Singh et al., 2011) and also using neutron transport (Lathouwers, 2003; Gupta and Modak, 2011).

The γ-modes were presented in (Ronen et al., 1976 and Velarde et al., 1978), but as far as the authors know, there is not a lot of work devoted to this kind of modes. Recently, (Avvakumov et al., 2017) a new spectral problem has been formulated (δ-modes), which is connected to self-adjoint part of operator of neutron absorption-generation to make an a priori estimate of neutron flux dynamics.

Different methods have been proposed to discretize the neutron diffusion equation. Modern nodal methods usually rely in the Nodal Expansion Method (NEM) (Finnemann, 1975; Singh et al., 2014) and analytical nodal method (ANM) (Smith, 1979; Hébert, 1987). Also, nodal collocation methods have been used to study reactors with rectangular geometries (Verdú et al., 1994). h-p high order finite elements methods have also developed using two refinement techniques: a subdivision of the spatial mesh (h-refinement) and also the possibility of increasing the polynomial degree used in the finite element expansions (Wang et al., 2009). In this work, to discretize the different modes equations, a high order finite element method similar to the one presented in (Vidal-Ferrandiz et al., 2014) is used.

Generally, the dominant (or the smallest) eigenvalue and its corresponding eigenfunction are computed to study the criticality of reactor and to know the steady state neutron distribution in the core. Next eigenvalues are interesting because they have been successfully used to develop modal methods and to classify and study neutronic oscillations (Miró et al., 2002). Thus, it will be interesting

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http://dx.doi.org/10.1016/j.anucene.2017.08.018
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to develop efficient methods to compute a set of eigenpairs, not just the first one. For that purpose, Krylov subspace based methods have shown effective, (Verdú et al., 1999; Verdú et al., 2005). Nevertheless, to compute the dominant modes of a reactor for different configurations, block-Newton methods have shown to be very efficient (Lösche et al., 1998; González-Pintor et al., 2011). These authors have studied a block-Newton method to solve ordinary eigenvalue problems. Since the different modes equations are expressed as generalized eigenvalue problems, in this work, two new extensions of the block-Newton method for generalized eigenvalue problems are proposed.

The main aim of this paper is to obtain, using a finite element method, the discrete eigenvalue problems associated with the λ, γ and α-modes, of the neutron diffusion equation to compare these modes and to analyze different strategies to compute them for a given configuration of a nuclear power reactor, combining the Krylov–Schur method (Stewart, 2002) and the block-Newton methods for generalized eigenvalue problems.

The structure of the rest of the paper is as follows. In Section 2, the λ, γ and α modes are defined for the neutron diffusion equation in the approximation of two energy groups. Also, in this section, the relations between these modes are analyzed. In Section 3, the spatial discretization used for the modes equations is briefly presented. In Section 4, the description of eigensolvers used to solve the algebraic problems obtained with the discretization is given. Numerical results for the analysis of two different kind of reactors are presented in Section 5. Finally, the main conclusions of the paper are summarized in Section 6.

2. Definition of spatial modes

The spatial modes problems are obtained from the two energy groups approximation of the neutron diffusion equation, however the formulations obtained can be easily extended to any number of groups of energy.

The time dependent neutron diffusion equation with K groups of delayed neutron precursors is of the form (Stacey, 2007)

$$\begin{aligned} V^{-1}\frac{\partial \phi}{\partial t} + \mathcal{L}\phi + \mathcal{S}\phi &= (1-\beta)\mathcal{F}\phi + \sum_{k=1}^{K}\lambda_k^d \mathcal{C}_k \chi, \\ \frac{\mathrm{d}\mathcal{C}_k}{\mathrm{d}t} &= \beta_k \mathcal{F}_1 \phi - \lambda_k^d \mathcal{C}_k, \qquad k = 1,\ldots,K, \end{aligned} \tag{1}$$

where

$$\begin{aligned} \mathcal{L} &= \begin{pmatrix} -\vec{\nabla}(D_1\vec{\nabla}) + \Sigma_{a_1} + \Sigma_{12} & 0 \\ 0 & -\vec{\nabla}(D_2\vec{\nabla}) + \Sigma_{a_2} \end{pmatrix}, \\ \mathcal{S} &= \begin{pmatrix} 0 & 0 \\ -\Sigma_{12} & 0 \end{pmatrix}, \qquad \mathcal{F} = \begin{pmatrix} \nu\Sigma_{f_1} & \nu\Sigma_{f_2} \\ 0 & 0 \end{pmatrix}, \\ V^{-1} &= \begin{pmatrix} \frac{1}{v_1} & 0 \\ 0 & \frac{1}{v_2} \end{pmatrix}, \qquad \chi = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad \phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}, \\ \mathcal{F}_1 &= \begin{pmatrix} \nu\Sigma_{f_1} & \nu\Sigma_{f_2} \end{pmatrix}. \end{aligned} \tag{2}$$

Eq. (1) can be transformed into several time-independent eigenvalue problems. Thus, criticality is forced of several forms obtained different eigenvalue problems: the λ and the γ-modes problems. Assuming that the time dependence of the neutron flux has an exponential behaviour the α-modes problem is obtained.

If the fission nuclear cross sections are divided by a positive number, λ, the following steady-state equations are obtained

$$\begin{aligned} \mathcal{L}\psi + \mathcal{S}\psi &= (1-\beta)\frac{\mathcal{F}}{\lambda}\psi + \sum_{k=1}^{K}\lambda_k^d \mathcal{C}_k \chi, \\ 0 &= \beta_k \frac{\mathcal{F}}{\lambda}\psi - \lambda_k^d \mathcal{C}_k \chi, \qquad k = 1,\ldots,K, \end{aligned} \tag{3}$$

that is,

$$\mathcal{L}\psi + \mathcal{S}\psi = (1-\beta)\frac{\mathcal{F}}{\lambda}\psi + \sum_{k=1}^{K}\beta_k \frac{\mathcal{F}}{\lambda}\psi. \tag{4}$$

Taking into account that $\sum_{k=1}^{K}\beta_k = \beta$, the λ-modes problem is obtained,

$$(\mathcal{L}+\mathcal{S})\psi_n = \frac{1}{\lambda_n}\mathcal{F}\psi_n. \tag{5}$$

The adjoint problem associated with the λ-modes is given by

$$(\mathcal{L}^\dagger + \mathcal{S}^\dagger)\psi_n^\dagger = \frac{1}{\lambda_n}\mathcal{F}^\dagger\psi_n^\dagger, \tag{6}$$

where $\mathcal{L}^\dagger, \mathcal{S}^\dagger$ and $\mathcal{F}^\dagger$ are the transpose operators of $\mathcal{L}, \mathcal{S}$ and $\mathcal{F}$, respectively.

The λ-modes, ψ_n, and the adjoint λ-modes, $\psi_m^\dagger$, satisfy the biorthogonality relation

$$\langle \psi_m^\dagger, \mathcal{F}\psi_n \rangle = \int_\Omega dV\,(\psi_m^\dagger)^T \mathcal{F}\psi_n = \delta_{m,n}\langle \psi_n^\dagger, \mathcal{F}\psi_n \rangle, \tag{7}$$

where Ω is the volume defined by the reactor core and $\delta_{m,n}$ is the Kronecker's delta.

If the fission and scattering terms of (1) are divided by $\gamma > 0$ to obtain the steady-state equations, a process similar to the one used to obtain the λ-modes can be followed to obtain the γ-modes problem, which has the following form

$$\mathcal{L}\phi_n = \frac{1}{\gamma_n}(\mathcal{F}-\mathcal{S})\phi_n. \tag{8}$$

It is possible to obtain a relation between the λ-modes and the γ-modes in terms of the adjoint λ-modes problem (6). We start multiplying the Eq. (8) by the adjoint λ-mode, $\psi_n^\dagger$, and integrating over the domain, obtaining

$$\langle \psi_n^\dagger, \mathcal{L}\phi_n \rangle = \left\langle \psi_n^\dagger, \frac{1}{\gamma_n}(\mathcal{F}-\mathcal{S})\phi_n \right\rangle, \tag{9}$$

or by symmetry of $\mathcal{L}$,

$$\langle \mathcal{L}\psi_n^\dagger, \phi_n \rangle = \left\langle \psi_n^\dagger, \frac{1}{\gamma_n}(\mathcal{F}-\mathcal{S})\phi_n \right\rangle. \tag{10}$$

By Eq. (6),

$$\mathcal{L}\psi_n^\dagger = \frac{1}{\lambda_n}\mathcal{F}^\dagger\psi_n^\dagger - \mathcal{S}^\dagger\psi_n^\dagger, \tag{11}$$

thus, Eq. (10) is equivalent to

$$\frac{1}{\lambda_n}\langle \mathcal{F}^\dagger\psi_n^\dagger, \phi_n \rangle = \frac{1}{\gamma_n}\langle \psi_n^\dagger, (\mathcal{F}-\mathcal{S})\phi_n \rangle + \langle \psi_n^\dagger, \mathcal{S}\phi_n \rangle. \tag{12}$$

Simplifying and isolating λ_n in (12), we have

$$\frac{1}{\lambda_n} = \frac{1}{\gamma_n} + \left(1 - \frac{1}{\gamma_n}\right)\frac{\langle \psi_n^\dagger, \mathcal{S}\phi_n \rangle}{\langle \psi_n^\dagger, \mathcal{F}\phi_n \rangle}. \tag{13}$$

To obtain the intermediate α-modes, we consider again the neutron diffusion Eq. (1) where the delayed neutron precursors are assumed to be in steady state, that is

$$0 = \beta_k \mathcal{F}_1 \phi - \lambda_k^d \mathcal{C}_k, \qquad k = 1,\ldots,K. \tag{14}$$

Other treatment of the neutron precursors lead to the prompt or total α-modes (Verdu et al., 2010).

This equality and the definition of β imply that

$$V^{-1}\frac{\partial \phi}{\partial t} + \mathcal{L}\phi + \mathcal{S}\phi = \mathcal{F}\phi. \tag{15}$$

Assuming that the neutronic flux admits a factorization