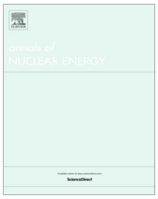
Annals of Nuclear Energy 109 (2017) 227–236

Contents lists available at ScienceDirect

Annals of Nuclear Energy

journal homepage: www.elsevier.com/locate/anucene



Finite volume lattice Boltzmann scheme for neutron/radiative transfer on unstructured mesh

Yahui Wang [a], Liming Yan [a], Yu Ma [a,*], Wei Li [b,*]

[a] School of Energy Science and Engineering, Harbin Institute of Technology, Harbin 150001, PR China
[b] Department of Mathematics & Statistics, Old Dominion University, Norfolk, VA 23529, USA

ARTICLE INFO

Article history:
Received 19 October 2016
Received in revised form 8 January 2017
Accepted 9 May 2017

Keywords:
Neutron/radiative transfer
Finite-volume lattice Boltzmann
Unstructured mesh
Boltzmann transport equation

ABSTRACT

In this paper, we propose a novel finite volume lattice Boltzmann (FV-LB) scheme to simulate the neutron and radiative transfer problems governed by the linear Boltzmann transport equation (BTE). In the derivation of the FV-LB scheme, we utilize a multi-speed lattice Boltzmann model to reduce the linear BTE into a linear discrete velocity Boltzmann equation (DVBE). Considering the computational effectiveness and flexibility in complex geometries, we design a finite volume scheme by using the first order upwind numerical flux and solve the DVBE with the finite volume scheme on a unstructured tetrahedral or triangular mesh. The accuracy of the FV-LB scheme is verified by comparing our results with the benchmark values of three representative problems.

1. Introduction

Neutron transport and radiative transfer have been active areas of research ever since the emergence of the technical concepts. Experimental investigations and numerical simulations about the two processes have received considerable attention due to their wide spectrum of applications: from nuclear reactor design (Mahadevan et al., 2014), nuclear reactor engineering (Glasstone and Sesonske, 1980) and neutron transport physicists (Benoist, 2015) to radiative transfer specialists (Schuster, 1903), optical tomography (Rahman and Rahman, 2015), short-pulsed laser in materials (Bhowmik et al., 2014), astrophysics scientists (Modest, 2003), laser therapy (Pietro et al., 2014) and other applications. Theoretically, neutron transport and radiative transfer problems share a common governing equation, namely, the linear Boltzmann transport equation (BTE) (Lewis and Miller, 1983). Compared with the domain interaction between neutrons/particles and the background atoms, the inter-particle collisions are negligible, thus only affine source term appeared in the RHS of the general BTE and the nonlinear term due to binary inter-particle collision is vanished (Bindra and Patil, 2012). Nevertheless, the linear BTE is still a tricky multi-dimensional (one for time, two or three for space, one or two for angle and one for energy or spectrum) integro-differential equation, whose analytical solutions are rarely obtained only with some very simplified conditions. Therefore, numerical simulation of the neutron/radiative transfer problem has been widely adopted and the simulation techniques have been a topic of research for more than a half century (Azmy and Sartori, 2010).

Numerical techniques for solving the neutron and radiative transfer problems can be divided into two categories, including stochastic approaches and deterministic methods. The stochastic approaches are rooted in the Lagrangian specification of the field. These techniques use sampling and stochastic numerical experiments on a swarm of particles to mimic the transport process. Among the most frequently adopted stochastic approaches in the transient neutron and the radiative transfer simulation is the Monte Carlo method (MCM) (Pang et al., 2016). MCM tracks the trajectory of each individual particle parcel, including its travels in the seven-dimensional space and occasionally collisions controlled by stochastic rule with the background atoms. With MCM, each macroscopic quantity is based on corresponding statistical averaging of the representative neutrons and particles microscopic quantity. This feature causes two mutually contradictory disadvantages of MCM: statistical error (MORI et al., 2012) and high computational cost. Recently, several improvements have been attempted in the MCM community to mitigate these disadvantages. Wang et al. (2014) developed Reactor Monte Carlo for reactor physics analysis on high-performance computing platforms (HPC). Lu et al., (Vol., 2004) proposed reverse Monte Carlo method for the radiative transfer problem to enhance efficiency and speedup computation. Martinelli et al. (2011) derived the scaling relationships required in the applications of the single MCM and Song et al. (2014) implemented a CAD-based superMC with the ITER bench-

---

* Corresponding authors.
E-mail addresses: mayu@hit.edu.cn (Y. Ma), wlixx007@odu.edu (W. Li).

http://dx.doi.org/10.1016/j.anucene.2017.05.022

Nomenclature

$\boldsymbol{e}$	lattice speed, cm/s
i, j, k	index of control volume
M	total number of discrete directions
$\boldsymbol{n}$	unit outward normal vector, cm
Q	total source, n/(cm^3 s)
$\boldsymbol{r}$	position vector, cm
S	area of interface, cm
t	time, s
v	speed of neutron/particle, cm/s
V	control volume, cm^2

Greek symbols

ε	non-dimensional parameter for Chapman-Enskog expansion
θ	zenith angle
$\boldsymbol{\Theta}$	total source term of the lattice Boltzmann model, n/(cm^2 s^2)
Σ_t, Σ_s	extinction coefficient, scattering coefficient, cm^{-1}
τ_c	relaxation time, s
φ	azimuth angle
ϕ	distribution function, n/(cm^2 s)
$\boldsymbol{\Phi}$	scalar flux, n/(cm^2 s)
ψ	directional flux, n/(cm^2 s)
$\boldsymbol{\Omega}$	angular direction, (sr)

Subscripts

s	scattering
t	total
α, α'	index for direction

Superscripts

eq	equilibrium

mark model. In contrast to the stochastic methods, deterministic methods for the neutron and radiative transfer problems are based on the Eulerian specification of the field. They convert the linear BTE into simpler equations through various dimension reduction techniques and solve the resultant equation using conventional numerical techniques. With certain simple boundary conditions, one can reduce the angle and the energy/spectrum dimensions in the linear BTE by integration, ending up with an integral equation (IE) and then solve the IE with more convenient numerical methods (Tan and Hsu, 2001). With general boundary conditions, one can apply the discrete ordinate method (DOM) (Ganapol, 2015) to transform the linear BTE into a simultaneous set of partial differential equations (PDEs) and solve them using mature numerical PDE techniques, such as the method of characteristic (MOC) (Hoffman and Lee, 2016), the finite volume method (FVM) (Chai et al., 1994) the finite difference method (FDM) (Zhu et al., 2016) or the finite element method (FEM) (Mercimek and Özgener, 2014). Furthermore, the radiative transfer and the neutron transport communities have respective preferences for methods and nomenclatures. The discrete transfer method (DTM) (Nirgudkar et al., 2014), and the collapsed dimensional method (CDM) (Talukdar, 2006) are widely used in radiative transfer, whereas the DOM is popular in both neutron transport and radiative transfer.

The lattice Boltzmann method (LBM) for neutron and radiative transfer problems falls in the category of deterministic methods. Although LBM was originally regarded as a Navier-Stokes solver, it was soon known that the BTE can be simulated by LBM as well if more discrete velocities in the phase-space are included in the model, namely the multi-speed LBM model. Over the years, researchers have found LBM a popular choice of solving the BTE with BGK simplification (Qian et al., 1992) due to the attractive properties of LBM, like the simple construction, the mesoscopic nature, the ability to handle complex geometry and the inherent parallel nature (Chen and Doolen, 1998; Succi, 2001). Accordingly, LBM has been applied to solve various problems, such as heat and mass transfer (Wang et al., 2007), multi-phase flow (Házi et al., 2002), magneto hydrodynamic (Sheikholeslami et al., 2014), phonon transport (Guo and Wang, 2016) and combustion (Filippova and Hänel, 2000). In the application of the radiative heat transfer problem, a hybrid method is usually adopted: the LBM solves the heat transfer and a conventional method such as the DOM (Mishra et al., 2011) or the FVM (Das et al., 2011) simulates the radiative transfer process. More recently, several LBM schemes especially for the radiative transfer problem (Asinari et al., 2010; Bindra and Patil, 2012; Ma et al., 2011; Rienzo et al., 2011; Zhang et al., 2013) were designed. Mishra et al., (Asinari et al., 2010) developed an LBM scheme for solving the benchmark radiative equilibrium problem involve a two-dimensional rectangular enclosure and the results show excellent agreement with those from the FVM. Ma et al. (2011) proposed another LBM model for one-dimensional radiative transfer by using the Chapman-Enskog expansion. Based on the work of Ma et al., Bindra and Patil (2012) extended the LBM model to solving the radiative or neutron transport equation with the scattering term. In addition, Zhang et al. (2013) investigated the LBM for solving the short-pulsed laser propagation in a participating slab.

However, the current researches of LBM for the neutron/radiative transfer problems didn't take into account the complex geometry in reality. The reviewed LBE simulations are implemented on uniform structured meshes. In complex geometry, generating high quality body-fitted structured mesh is difficult. A natural idea to overcome this obstacle is to use unstructured mesh. In this paper, we present a novel finite-volume lattice Boltzmann (FV-LB) scheme to solve the transient neutron and radiative transfer problems on unstructured mesh.

The rest of this paper is organized as follows. In Section 2, the detailed derivation of the FV-LB scheme for the neutron/radiative transfer is discussed, including the discretization of the linear BTE, the finite volume scheme for the linear discrete velocity Boltzmann equation (DVBE), the treatment of boundary conditions and the outline of the numerical implementation. In Section 3, we implement the simulations of three representative problems. The results are discussed and compared with benchmark values to verify the effectiveness and accuracy of the present FV-LB scheme. Section 4 is the conclusion of this paper.

2. Mathematical formulation

In this section, we first discretize the angular space of the linear BTE for the neutron and radiative transfer problems to obtain the DVBE. Then, we present the FV-LB scheme to solve the DVBE on an unstructured tetrahedral mesh. The treatments of representatively vacuum and reflective boundary conditions are addressed subsequently. The implementation procedure of the FV-LB scheme is listed in the rest of this section.