Annals of Nuclear Energy 109 (2017) 284–297

Contents lists available at ScienceDirect

Annals of Nuclear Energy

journal homepage: www.elsevier.com/locate/anucene

Use cases of Tucker decomposition method for reconstruction of neutron macroscopic cross-sections

Thi Hieu Luu [a,b,*], Matthieu Guillo [a], Yvon Maday [b,c], Pierre Guérin [a]

[a] Département SINETICS, EDF Lab Paris-Saclay 7, Boulevard Gaspard Monge, 91120 Palaiseau, France
[b] Sorbonne Universités, UPMC Univ Paris 06, UMR 7598, Laboratoire Jacques-Louis Lions, F-75005 Paris, France
[c] Institut Universitaire de France and Division of Applied Mathematics, Brown University, Providence, RI, USA

ARTICLE INFO

Article history:
Received 8 November 2016
Received in revised form 15 March 2017
Accepted 10 May 2017

Keywords:
Cross-section reconstruction
Tucker decomposition
Interpolation
Low-rank tensor approximation
Statistical analysis

ABSTRACT

The neutron cross-sections are inputs for nuclear reactor core simulations, they depend on various physical parameters. Because of industrial constraint (e.g. calculation time), the cross-sections can not be calculated on the fly due to the huge number of them. Hence, a reconstruction (or interpolation) process is used in order to evaluate the cross-sections at every point required, from (as few as possible) pre-calculated points. With most classical methods (for example: multilinear interpolation which is used in the core code COCAGNE of EDF (Électricité De France)), high accuracy for the reconstruction often requires a lot of pre-calculated points. We propose to use the Tucker decomposition, a low-rank tensor approximation method, to deal with this problem. The Tucker decomposition allows us to capture the most important information (one parameter at a time) to reconstruct the cross-sections. This information is stored as basis functions (called *tensor directional basis functions*) and the *coefficients* of the decomposition instead of pre-calculated points. Full reconstruction is done at the core code level using these decompositions. In this paper, a simplified multivariate analysis technique (based on statistical analysis) is also proposed in order to demonstrate that we can improve the quality of the acquired information as well as the accuracy of our approach. Using the Tucker decomposition, we will show in proposed use cases that we can reduce significantly the number of pre-calculated points and the storage size (compared to the multilinear interpolations) while achieving high accuracy for the reconstruction, even on a larger domain of parameters.

1. Introduction

Like most companies working in nuclear electricity production, Électricité De France in its research and development (EDF R&D) departments develops highly accurate nuclear reactor core simulator system. Two main classes of approach are employed for simulations: deterministic and probabilistic. Our work relates to the deterministic one.

The purpose of a core simulator system is to be able to simulate any kind of physical quantity for the proper operation and the safety of the power plant. In order to do that, one has to solve Boltzmann's equations (or an approximation of these) for neutrons. These equations need, at every (physical-) cell of the 3D space, some physical inputs, the cross-sections, denoted by the letter Σ, that model the interactions between fission-induced neutrons and nuclei from either the fuel or the moderator (water in the case of PWR).

These cross-sections vary from one (physical-) point of the core to another - hence $\Sigma = \Sigma(\vec{P})$ with $\vec{P} = (x, y, z)$ – and depend on d local parameters (so called feedback parameters), such as: *burnup (bu), fuel temperature (t_f), moderator density (ρ_m), boron concentration (b_c), xenon level (xe), etc.* Each feedback parameter can be seen as one axis of a d-dimensional space called *parameter-phase space* and a set of parameter values as a point in this parameter-phase space. Therefore, for each cell in a 3D space, there is a d-tuple parameter's value in the d-dimensional parameter-phase space. The actual value of d is model dependent.

In order to do that, core simulator systems classically involve two solvers used in chain: a lattice code and a core code. At the EDF in the department SINETICS (SImulation NEutronique, Technologie de l'Information, Calcul Scientifique), we use, in a first step, the lattice code APOLLO2 (Sanchez et al., 1988, 2010) developed at CEA, that generates cross-sections in the parameter-phase space

* Corresponding author at: Département SINETICS, EDF Lab Paris-Saclay 7, Boulevard Gaspard Monge, 91120 Palaiseau, France.
E-mail addresses: luu@ljll.math.upmc.fr (T.H. Luu), matthieu.guillo@edf.fr (M. Guillo), maday@ann.jussieu.fr (Y. Maday), pierre.guerin@edf.fr (P. Guérin).

http://dx.doi.org/10.1016/j.anucene.2017.05.032
0306-4549/

and store them inside a file. This file acts like a database; we call it a "*nuclear library*". Then, in a second step, the core code COCAGNE (Plagne et al., 2005) reads this library and, for every cell in the 3D space, computes the values of the d parameters (using for instance thermal–hydraulic code, depletion loop and so on) and evaluates the values of the cross-sections using the database and a reconstruction model. The reconstruction allows from values in the nuclear library (the smaller in size the better) to get an approximate value at any point in the parameter-phase space (the more accurate the better).

For the simulation of nuclear reactor core, one generally defines a particular point in the parameter-phase space at which the reactor core operates normally. This point is called *nominal point/condition*. Actually, the *burnup* direction is not involved in the definition of the nominal point since it is related to time. The domain near the nominal point (which thus lives in $\mathbb{R}^{d-1}$) is referred to as the *standard domain*. It is composed of values of the parameters in the parameter-phase space that are encountered under standard working conditions of the power plant. On the contrary, it is called the *extended domain* when the parameters get out of these running situations and are encountered in some special operations for the reactor or incidental situations. There is no much precise definition of these domains that may represent different objects through various publications.

In our proposed use cases, the standard domain and the extended domains are hypercube in $\mathbb{R}^5$. Here, the standard domain is a subset of extended domains as is precisely presented in Section 5.3 and approximately presented in Section 5.4.

In general, high accuracy for the reconstruction requires a lot of pre-calculated points, i.e., a lot of lattice calculations. The total number of pre-calculations performed by the lattice code is usually so large (∼thousands) that it takes a lot of calculation time (while a calculation takes about 30 s). This becomes more cumbersome for an extended domain with a model such as multilinear interpolation. The reduction of the number of these calculations as much as possible has been the motivation for introducing in Luu et al. (2017) a new reconstruction based on a low-rank tensor approximation method, that is referred to as *the Tucker decomposition*. The purpose of the current paper is to present and test on various assemblies (UOX, UOX-Gd, MOX, on standard and extended domains) this new reconstruction and compare it to the currently used multi-linear interpolator in COCAGNE.

In this paper, we also propose to use some new techniques, such as: the Empirical Interpolation Method (EIM) and the pre/post-analysis in order to achieve better accuracy for our method, compared to results presented in the paper (Luu et al., 2015). Moreover, results of new use cases (performed on extended domains) will be shown.

The paper is organized in the following manner:

- Section 2 gives an overview of different methods for the reconstruction of cross-sections used by different utility companies.
- In Section 3, we present the Tucker decomposition applied to a set of multivariate functions.
- Section 4 illustrates how we can efficiently use the Tucker decomposition for the cross-section reconstruction problem and how we can deduce cross-section properties in high dimension. In this section, we will show the advantages of our model, which allow us to pre-analyze and post-analyze our approach.
- Section 5 is reserved to our proposed use cases, applied to different fuel assemblies (UOX, UOX-Gd, MOX). The results obtained demonstrate that we can reduce the pre-calculated data while achieving high accuracy in both cases: the standard domain and the extended domain. We also present some problems that we encountered in this section.
- Section 6 is dedicated to conclusion and perspective.

2. Overview of different methods for the reconstruction of neutron cross-sections

There are many core simulator systems for nuclear reactor simulations, for example: the pair of softwares ARCADIA(HERMES)-ARTEMIS (Hobson et al., 2013) used by AREVA, DRAGON-DONJON (Polytechnique Montréal and DRAGON code, 2016) used by Canadian Nuclear Society, NEXUS-ANC (Müller et al., 2007) used by Westinghouse, etc. Each such system employs a model in order to reconstruct the cross-sections. These models can be classified in two main categories:

- The cross-section values are approximated by adding some perturbation or correction terms to values calculated at or around the nominal point, (see e.g. Fujita et al., 2014; Turski et al., 1997; Müller et al., 2007; Stålek and Demazière, 2008). In general, the correction terms are based on physical knowledge or some expansion techniques, such as the Taylor expansion.
- The cross-section values are interpolated from the pre-calculated values on a grid (for instance, Cartesian grid constructed with a tensor product of the discretized points on each axis (Watson and Ivanov, 2002), sparse grid (Danniëll and Pavel, 2014), quasi-random grid (Dufek, 2011), etc.). The interpolation techniques in these models are different by the choice of basis functions: piece-wise linear functions, B-spline, Lagrange polynomials, etc. COCAGNE belongs to this category where the Cartesian grid and the piece-wise linear functions are used in the multilinear interpolation model.

The first category's methods have been proven suitable for the simulation on a "standard" domain where the parameter-values are rather close to the nominal condition. Of course, when they are far away from this condition, their accuracy is lost since the heuristics used around nominal values may not be valid anymore. The second ones are more general but high accuracy requires a lot of pre-calculated data. For instance, the multilinear approach that is used at EDF, relies on the acquisition of the values of the various cross sections on a Cartesian grid of the (standard or extended) domain in the parameter-phase space, i.e. in $\mathbb{R}^5$ from the lattice code. This nuclear library has a cardinal equal to N^d, where N stands for the number of discretized points in each phase direction. Then the reconstruction allows us to build an approximation of each cross section at any point by: i) locating this point to one of the cells of the Cartesian grid (to which this point belongs), here each cell is seen as a d dimensional object and its vertices are the points on which the value of each cross section is available, ii) averaging the previous values in a convex way to provide a linear approximation in each dimension. This is a very simple methods, and its accuracy (second oder in L^2 or L^∞ norms scaling like $\mathcal{O}(N^{-2})$) in terms of size of the cell that implies to have a quite large library on standard domains and very - even too much - large nuclear library on extended domains.

Therefore, either we have a high efficiency reconstruction method (meaning high accuracy with few points) but on a specific domain only, or we have an expensive reconstruction method (meaning high accuracy at a lot of pre-calculated points) but suitable for any domain.