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Preliminary study of applying adjoint-based mesh optimization method to nuclear power plant safety analysis

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ABSTRACT

Although the number of meshes and their distribution are important parameters that affect both computation resources and the accuracy of the numerical results, meshing of most of all computational problems still depends highly on the users' experiences. In this paper, a deterministic adjoint-based method of optimizing mesh distribution is proposed. The developed method is applied to a nuclear power plant safety analysis. The mesh optimization was performed with 1D steady state cylindrical nuclear fuel problem first. Radial and axial mesh distributions are optimized respectively. With no surprise, the optimized mesh system performs superior than the same number of uniformly meshed system. However, it was unexpected that the optimized mesh retains generality and therefore, the optimized mesh system can be still the best mesh system for given number of meshes under different condition or even during transient analysis. The authors applied the optimized mesh distribution to nuclear system safety analysis. A large pressurized water reactor cold leg guillotine break (LBLOCA) scenario was analyzed and the consequence of different mesh systems is investigated and discussed. From this preliminary study the usefulness and implication of the adjoint based mesh optimization method for the nuclear safety analysis is uncovered.

1. Introduction

Nowadays, research works for improving the accuracy and reducing the user-to-user variations for nuclear system analysis codes, such as MARS-KS (Lee, 2007) RELAP-5 (Nuclear Regulatory Commission, 2001) COBRA (Thurgood et al., 1983), are widely conducted. As a part of the effort, there are attempts to utilize the global optimization methods such as genetic algorithms (Tsai et al., 2014; Marseguerra and Zio, 2001; Marseguerra et al., 2003; Marseguerra et al., 2004) on many user-defined-parameters (UDPs) to reduce uncertainties for using the nuclear system analysis codes.

System analysis codes typically solve discretized partial differential equations by using a method of uniform or empirical meshing method based on user's judgement solely (Chung et al., 2010). Meshing still depends highly on the users' experiences, although the number of nodes and size of each node are important parameters that affect both computation resources and the accuracy of the results. However, if we can optimize the node distribution that can compute the most precise numerical solution of some problems under limited computing resource, positive effects such as increment of numerical stability, minimized user dependency, and increased reliability of the system analysis code can be expected.

In this study, a method for optimizing mesh distribution is proposed. The proposed method uses adjoint base optimization method (adjoint method) Errico, 1997; Cao et al., 2003 which is one of the global optimization techniques. Furthermore, by applying the proposed methodology to a 1-D steady state cylindrical nuclear fuel rod, the authors confirmed whether the suggested method can give more accurate solution than that of the conventional method of uniform meshing produces. As well, the optimized result will be obtained by applying this meshing technique to the existing code and will be compared to the results produced from the uniform meshing method.

The developed method is applied to a nuclear fuel representing the average core. The fuel is cylindrical shape and it is consisted of fuel pellet, gap and cladding with uniform volumetric heat generation. The boundary condition is given as the coolant temperature. Numerical solutions are calculated from an in-house 1D Finite Difference Method (FDM) code while neglecting the axial conduction. The fuel radial node optimization was first performed to match the Fuel Centerline Temperature (FCT) the best. This was followed by optimizing the axial node which the Peak Cladding Temperature (PCT) during steady state operation matches the best. After obtaining the optimized radial and axial nodes, the nodalization is imple-

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Nomenclature

Latin letters

C_p	heat capacity at constant pressure
q''	heat flux
q'''	volumetric heat generation
k	thermal conductivity
T	temperature
t	time
r	radial coordinate
z	axial coordinate
h	convection heat transfer coefficient
N	number of nodes
n	number of elements of subscripted vector
A	matrix
b	source vector
$\boldsymbol{x}$	field vector
$\boldsymbol{p}$	design parameter vector
g	object function
C	constant
$\boldsymbol{V}$	sensitivity vector
norm	second-order norm
max	maximum value
min	minimum value
abs	absolute value
O(*)	computational complexity of *

Greek letters

∂	partial difference
∇	del operator
∇^2	laplacian operator ($\nabla \cdot \nabla$)
Δ	difference
ρ	density
λ	Lagrange multiplier

Subscripts

gap	gap
fuel	fuel
Clad	cladding
Coolant	coolant
prev	previous
next	next
step	step
rf	relaxation factor
i, j	numbering

Superscripts

T	transpose matrix
-1	inverse matrix

mented into the system analysis code and transient analyses were performed to observe the optimum nodalization performance.

2. Methodology

2.1. Adjoint based optimization method

The gradient of an object function at the design variable space (sensitivity vector) is useful information for optimization. One way to find the sensitivity vector is to compute differences over the all elements of design variables. If original problem is $g(\boldsymbol{x},\boldsymbol{p})$ (g: object function, $\boldsymbol{x}$: field vector. $\boldsymbol{p}$: design variable vector), the sensitivity vector $\frac{dg(\boldsymbol{x},\boldsymbol{p})}{d\boldsymbol{p}}$ can be obtained numerically (See Eq. (1)).

$$\frac{dg(\boldsymbol{x},p_i)}{dp_i} \approx \frac{g(\boldsymbol{x},p_i+p_i)-g(\boldsymbol{x},p_i)}{p_i}, p_i \in \boldsymbol{p} \tag{1}$$

By this way, $g(\boldsymbol{x},p_i+\Delta p_i)$ for every $p_i \in \boldsymbol{p}$ is required to get the full sensitivity vector. This is equivalent to solving the original problem $n\boldsymbol{p}$ times. For large $n\boldsymbol{p}$, it is considerably very costly.

In this paper, the authors are proposing to use an adjoint based optimization method. (adjoint method) Errico, 1997; Cao et al., 2003 The adjoint method is a technique to obtain the sensitivity vector with marginal increase in the computational cost of solving the original problem.

Sensitivity vector is given as Eq. (2), where $\boldsymbol{p}$ is the design variable vector, $g(\boldsymbol{x},\boldsymbol{p})$ is the object function and $\boldsymbol{x}$ satisfies $A\boldsymbol{x}=\boldsymbol{b}$. Dimension of $\boldsymbol{x}$, $\boldsymbol{b}$, $\boldsymbol{p}$, A are (1, M), (M, 1), (1, P) and (M, M) are

$$\frac{dg(\boldsymbol{x},\boldsymbol{p})}{d\boldsymbol{p}} = g_{\boldsymbol{p}} + g_{\boldsymbol{x}}x_{\boldsymbol{p}} \tag{2}$$

$$\left(A_{\boldsymbol{p}} = \left(\frac{\partial A}{\partial p_i}\right),\ i \in \boldsymbol{p}\right)$$

$$g_{\boldsymbol{x}}x_{\boldsymbol{p}} = g_{\boldsymbol{x}}[A^{-1}(\boldsymbol{b}_{\boldsymbol{p}} - A_{\boldsymbol{p}}\boldsymbol{x})] \tag{3}$$

$$(\boldsymbol{b}_{\boldsymbol{p}} = (\frac{\partial b}{\partial p_i}),\ i \in \boldsymbol{p})$$

Eq. (3) can be re-written as Eq. (4), which is the dual problem of Eq. (3).

$$g_{\boldsymbol{x}}x_{\boldsymbol{p}} = \lambda^T[(\boldsymbol{b}_{\boldsymbol{p}} - A_{\boldsymbol{p}}\boldsymbol{x})],\ A^T\lambda = g_{\boldsymbol{x}}^T \tag{4}$$

To obtain $\frac{dg(\boldsymbol{x},\boldsymbol{p})}{d\boldsymbol{p}}$, Eq. (4) requires only one inverse-matrix calculation. This is much less costly than the traditional calculation method of using the finite differences over the all elements of $\boldsymbol{p}$.

2.2. 1D FDM fuel rod numerical scheme

The problem can be modeled as shown in Fig. 1. The 1D nuclear fuel is composed of UO_2 fuel pellet with uniform heat generation, Zircaloy cladding, and the gap between them. There are four boundary and interfacial matching conditions: Centerline symmetric boundary, fuel-gap convection boundary, gap-cladding convection boundary, and cladding-coolant convection boundary. Coolant bulk temperature is specified for the radial mesh optimization.

The 1D heat conduction equation with the volumetric heat generation in a cylindrical geometry is shown in the following equation. (Incropera et al., 2013)

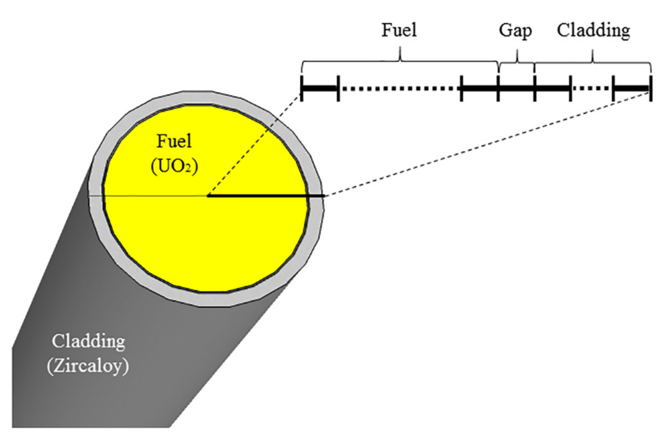

Fig. 1. 1D lumped fuel-cladding system problem concept picture.