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A semi-analytical treatment of xenon oscillations

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ABSTRACT

A novel approach is developed to investigate xenon oscillations within a two-group two-region coupled core reactor model incorporating thermal feedback and poison effects. Group-wise neutronic coupling coefficients between the core regions are calculated applying the associated fundamental and first mode eigenvalue separation values. The resultant nonlinear state space representation of the core behavior is quite suitable for evaluation of reactivity induced power transients such as load following operation. The model however comprises a multi-physics coupling of sub-systems with extremely distant relaxation times whose stiffness treatment inquire costly multistep implicit numerical methods. An adiabatic treatment of the sluggish poison dynamics is therefore proposed as a way out. The approach helps further investigate the nonlinear system within a linear time varying (LTV) framework whereby a semi-analytical framework is established. This scheme incorporates a matrix exponential analytical solution of the perturbed system as a quite efficient tool to study load following operation and control purposes. Poison dynamics are updated within larger intervals which exclude the need for specific numerical schemes of stiff systems. Simulation results of the axial offset conducted on a VVER-1000 reactor at the beginning (BOC) and the end of cycle (EOC) display quite acceptable results compared with available benchmarks. The LTV reactor model is further investigated within a stability analysis of the associated time varying systems at these two stages employing the concept of Lyapunov exponent.

1. Introduction

The projected increase in the share of nuclear energy for electricity production has brought load following related issues into active reconsideration (Adamantiades and Kessides, 2009). Development of appropriate kinetic models to investigate the dynamic behavior of nuclear reactors is therefore an ongoing research interest (Li et al., 2016). These schemes are however required to accompany sufficient details of the associated phenomena with an acceptable level of computational efficiency. Several space-time kinetic strategies with associated numerical routines which extend from simple lumped parameter to rigorous three dimensional approaches have been reported accordingly (Cho, 2005).

Dynamic behavior of a nuclear reactor core is determined by a set of coupled multi-physics phenomena with various relaxation timescales which call for careful numerical treatments (Zerkak et al., 2015). Neutron population varies almost immediately as a result of an initiating perturbation (e.g. control rod movement during load following operation). Delayed neutron precursors and thermal heat transfer feedbacks meanwhile affect the evolution of core power profile in longer timescales. Poison concentrations on the other hand change on an hourly basis. These conditions essentially become more pronounced in large PWR reactors with a low overall neutronic coupling within the core medium. Locally induced perturbations are thus not instantly propagated throughout the core of these large reactors (Obaidurrahman and Singh, 2010). This alongside with the intrinsically slow dynamics of xenon-iodine poison chain may entail specific instability issues in the reactor. Spatial power oscillations, upper and lower half flux tilt in the core and consequent displacement of the hot spot leading to serious safety issues are to name in this regard (Obaidurrahman and Singh, 2011).

Unwelcome convergence difficulties moreover occur throughout simulations as the quick nature of variations in the power (microseconds) are accompanied with the sluggish xenon dynamics (several hours). This common numerical stiffness feature is basically encountered while dealing with simultaneous multi-physics systems with widely spaced relaxation times whereby the effective system behavior is determined by the fastest evolving variable (Rahbar and Cady, 1981). Specific multistep numerical treatments (e.g. implicit Runge-Kutta or Gear method) have therefore been developed in the literature to overcome this difficulty

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(Seinfeld et al., 1970). These approaches however bear computational costs as they call for equal fine time meshes of all variables, a quite demanding task especially in long runs (e.g. load following simulation). A numerical efficient scheme whereby sufficient details are accompanied with relevant ease of implementation is therefore advisable in this regard.

The work presented herein is an attempt to efficiently alleviate the above mentioned concerns. An extended two region coupled reactor core model comprising xenon poisoning and thermal feedback effects is recast into a time varying system framework. Numerical aspects are investigated employing the analytical matrix exponential method (Yamoah et al., 2013) alongside with a quasi-static numerical treatment of the slow xenon dynamics (Saadi et al., 2010). This overall strategy quite efficiently eliminates the need for specific numerical integration schemes to overcome stiffness complexities. The paper is organized as follows: in the next section an overview of poison effects on reactor dynamics is presented and the need for a rather straightforward approach to the modeling of xenon oscillations within the core is discussed. The methodology to construct an appropriate two-group coupled-core reactor model which comprises poison and thermal feedbacks is presented in the section afterwards. This framework lays the basis for a semi-analytical treatment of the core behavior which will be accounted for in the fourth section. A discussion on the largest Lyapunov exponent criterion is further presented to investigate the stability behavior of xenon oscillations for the fresh (BOC) and depleted cores (EOC). This index provides a necessary measure for stability of the linear time variant (LTV) systems. Numerical results for a VVER-1000 reactor at these two stages are given in the last section whereby a comparison to benchmark (FSAR) results confirms further applicability of the model for load following simulation and related control purposes.

2. Xenon poisoning effects

The dynamic behavior of Xenon isotope as a highly poisoning byproduct of the fission process has long remained subject of several analytical investigations with quite diverse perspectives (Canosa and Brooks, 1966; Song and Cho, 1997; Zarei et al., 2016). This phenomenon is mainly attributed to the large absorption cross section of xenon for thermal neutrons ($\sigma_{a,Xe} \approx 2.5 \times 10^6\ barns$) accompanied by the small fission yield ($\gamma_X = 0.003$) and the effectively slow time dependent evolution of this isotope ($T_{1/2,Xe} \approx 9.1$ h). These distinct features lead to an overall delayed dynamic behavior to follow subscribed power maneuvers such as load following operation. A positive (negative) reactivity induced perturbation which causes power ascension (decrease) therefore entails an immediate reduction (increase) in xenon concentration to be further compensated several hours later through the β⁻ decay chain ($I^{135} \xrightarrow{T_{half} \approx 6.7\text{ h}} Xe^{135}$) of iodine ($\gamma_I = 0.061$). The situation is moreover complicated in large reactors with high a neutron flux ($\phi \approx 10^{13}$ neutrons/cm^2 s) (Lewins, 1979). As a result of the low neutronic couplings within these large cores, locally induced perturbations are propagated with a relative delay throughout the reactor (Obaidurrahman and Singh, 2011). Induced xenon and flux oscillations are therefore quite anticipatory. Appropriate modeling of the phenomena with an acceptable level of numerical efficiency for further load following control activities is desired in this regard.

3. The coupled core reactor model

Xenon induced oscillations are often investigated via a two-region layout of the core whereby flux tilt (i.e. upper and lower half power mismatch) is taken into account as a safety index (Obaidurrahman and Singh, 2011; Song and Cho, 1997). This framework provides a reasonable description of the axial offset of power (flux) and hot-spot displacement within the core as important safety related consequences thereof. A coupled two-region model (Baldwin, 1959; Kawai, 1965) will be accordingly retrofitted hereafter to comprise the underlying two-group neutron diffusion equation accompanied with xenon and thermal feedback effects. Eigenvalue separation value between the first and the fundamental mode of the diffusion equation ($L\phi = 1/\lambda F\phi$) is resorted to represent the underlying neutron flux coupling between the two half cores (Nishina and Tokashiki, 1996; Jospehson, 1978). This concept will be further elaborated to establish a framework for analytical description of the asymmetrical induced flux tilts within loosely coupled cores. Certain design data of the reactor core under scrutiny is given in Table 1.

Two slightly sub-critical half cores which constitute a whole critical system are neutronically coupled through establishment of an inter-zonal flux gradient as the neutron current (Fig. 1). While the fundamental mode eigenvalue ($\lambda|_{B_0=\pi/H}$) represents the tendency of neutron flux to satisfy whole core zero boundary conditions, the first harmonic ($\lambda|_{B_1=2\pi/H}$) implies a similar tendency for either half core (Jospehson, 1978).

The difference between these two eigenvalues thus reflects the contribution made by either half region to establish an overall critical system. A quite suitable reactivity measure is accordingly developed as in Eq. (1) for the intended coupled core coupling (Kawai, 1965; Nishina and Tokashiki, 1996).

$$\Delta\rho_{coupling} = \frac{1}{2}\left(\lambda|_{B=\pi/H} - \lambda|_{B=2\pi/H}\right) \tag{1}$$

This methodology is further applied upon a two-group layout of diffusion equation (Eq. (2)). Eigenvalue separation is calculated by setting $\det(M_I) = 0$ for the whole and half core buckling values respectively ($B = \pi/H, 2\pi/H$).

$$\begin{gathered}
\begin{cases} -D_1\nabla^2\phi_1 + \Sigma_{a1}\phi_1 + \Sigma_{s1\to2}\phi_1 = \frac{\upsilon(\Sigma_{f1}\phi_1 + \Sigma_{f2}\phi_2)}{k} \\ \qquad -D_2\nabla^2\phi_2 + \Sigma_{a2}\phi_2 = \Sigma_{s1\to2}\phi_1 \end{cases} \Rightarrow \\
M_I = \begin{bmatrix} D_1B^2 + \Sigma_{a1} + \Sigma_{s1\to2} - \frac{\upsilon\Sigma_{f1}\phi_1}{k} & -\frac{\Sigma_{f2}\phi_2}{k} \\ -\Sigma_{s1\to2} & D_2B^2 + \Sigma_{a2} \end{bmatrix} \\
\det(M_I) = 0 \xrightarrow{\begin{cases} B_0=\pi/H \\ B_1=2\pi/H \end{cases}} \begin{cases} k_0 \\ k_1 \end{cases} \Rightarrow \Delta\rho_{coupling} = \frac{1}{2}\left(\frac{1}{k_0} - \frac{1}{k_1}\right)
\end{gathered} \tag{2}$$

Table 1

Basic design data for VVER-1000 reactor.

Parameter	Value
Nominal Power (P_0)	3000 MW
Xenon value prior to perturbation (X_0)	$BOC = 9.0483 \times 10^{15}\,\#\ \text{cm}^{-3}$ $EOC = 1.0187 \times 10^{16}\,\#\ \text{cm}^{-3}$
Iodine fission yield (γ_I)	0.061
Iodine decay constant (λ_I)	$2.87 \times 10^{-5}\ \text{s}^{-1}$
Effective delayed neutron fraction (β_{eff})	700×10^{-5}
Effective precursor decay constant (λ_{eff})	8×10^{-2} s
Power feedback coefficient (α_f)	2.69×10^{-4} cm s
Average number of fission neutrons (υ)	2.45
Average speed of neutrons in the fast group (v_1)	3×10^7 cm/s
Average speed of neutrons in the thermal group (v_2)	2.2×10^5 cm/s
Energy released per fission (E_r)	200 MeV
Boron absorption cross section (σ_b)	760*barn*
Initial boric acid (H_3BO_3) concentration	6 g/l
Active core height (H)	355 cm
Active core diameter (d)	316 cm