Annals of Nuclear Energy 106 (2017) 154–159

Contents lists available at ScienceDirect

Annals of Nuclear Energy

journal homepage: www.elsevier.com/locate/anucene

The Monte Carlo GPT methodology for the analysis of ratios of functionals bilinear with the real and adjoint neutron fluxes

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ARTICLE INFO

Article history:
Received 15 December 2016
Received in revised form 28 March 2017
Accepted 29 March 2017
Available online 17 April 2017

Keywords:
Monte Carlo
Eranos
GPT
Sensitivity
Perturbation
Reactivity worths

ABSTRACT

Perturbation methods are part of the reactor physics foundation devoted to the study of fundamental quantities used in design and safety analysis of nuclear reactors. In deterministic codes, such as ERANOS, standard perturbation theory (SPT) and generalized perturbation theory (GPT) methods have been historically developed and used. Monte Carlo codes, such as MCNP 6.1, can also perform, via adjoint weighted tally, SPT calculations of reactivity worths. In this work a method, referred to as MC-GPT, is envisaged to enable Monte Carlo codes to be used also for GPT analysis. A preliminary comparison between calculations with MCNP and ERANOS relevant to perturbations affecting a given reactivity worth functional is presented and commented.

1. Introduction

As well known perturbation/sensitivity methods in the nuclear reactor field relevant to reactivity worth analysis, namely those associated with the Standard Perturbation Theory (SPT) (Wigner, 1945) and the Generalized Perturbation Theory (GPT) (Gandini, 1987), have been and are largely used for project and operation performance physical studies. These methods are implemented in most deterministic codes allowing multiple perturbation effect evaluations with faster than direct procedures. This is due to the fact that these methods, at first order, rather than the recalculation of the multiplication factor at perturbed conditions, imply simple integration operations in terms of unperturbed quantities. The new generation reactors (GEN IV) demand more stringent safety and no-proliferation requirements in the design stage. Such requirements often deal with reactor concepts with a high degree of core heterogeneity that are difficult to implement in deterministic neutronic transport codes.

In past years efforts have been made to implement perturbation techniques into Monte Carlo codes. Historically, MCNP 4B (McKinney, 1984) developed a perturbation module based on a differential techniques (Rief, 1984) which, by perturbing the track length estimator, allows perturbative analyses of reactivity on the collision estimator of k_{eff}. Such module has been extensively used by the authors in the analysis of an experimental champaign on the TAPIRO fast source reactor (Burgio et al. 2014). Recently, the Los Alamos Laboratory has implemented Monte Carlo estimators that lead to the evaluation of quantities associated with the SPT methodology into the MCNP 6.1 code (Denise Pelowitz, 2012) via a forward adjoint weighted method (Kiedrowski et al., 2011). A similar approach is suggested also by Nauchi and Kameyama (2010). At the moment, by the forward adjoint weighted tally algorithm, in MCNP point kinetic quantities and reactivity worths can be estimated whereas with deterministic codes, such as ERANOS (Rimpault et al., 2002), it is possible to extend the perturbation analysis to any functional of the real and/or adjoint neutron fluxes by using the GPT methodology, or its equivalent modality EGPT (Gandini et al., 1986). This latter modality has the advantage of implying solving homogeneous rather than inhomogeneous equations. Aim of this work is to explore the range of applicability of the EGPT methodology in the Monte Carlo MCNP code.

To be reminded also the implementation of perturbation techniques into the SERPENT code (Aufiero et al., 2015), which also allows to calculate sensitivity coefficients of a given quantity (response) with respect to the system parameters.

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http://dx.doi.org/10.1016/j.anucene.2017.03.044
0306-4549/

2. The EGPT method

The EGPT method applied to reactivity worth analysis is widely described in reference of Gandini, Palmiotti and Salvatores. We shall remind here its main formulations.

2.1. EGPT for reactivity worths

Consider as functional Q a generic reactivity worth ρ_c, as given by the expression, in the exact perturbation modality,

$$\rho_c = \frac{<\phi^* \delta_c B \phi_{(c)}>}{<\phi^* F_{(c)} \phi_{(c)}>} \tag{1}$$

where $\delta_c B = \delta_c A + \lambda\, \delta_c F$ and $\phi_{(c)}$ and $F_{(c)}$ indicate the neutron flux and the neutron production operator, respectively, at the state altered by the change $\delta_c B$ of B.

As can be easily verified, the above expression of ρ_c is equivalent to the following one:

$$\rho_c = \frac{<\phi^* \delta_c B_{(c)} \phi_{(c)}>}{<\phi^* F \phi_{(c)}>} \tag{2}$$

where $\delta_c B_{(c)} = \delta_c B + \delta_c \lambda \delta_c F \equiv \delta_c B - \rho_c \delta_c F$.

Considering the GPT methodology, the following perturbation expression may be written, after an alteration of system parameters implying a change $\delta_s B$ of the governing operator,[1]

$$(\delta Q)_{GPT} = \frac{<\psi^*_{c(c)} \delta_s B_{(c)} \phi_{(c)}>}{<\phi^* F_{(c)} \phi_{(c)}>} + \frac{<\phi^* \delta_s B \psi_c>}{<\phi^* F \phi_{(c)}>} - \rho_c \frac{<\phi^* \delta_s F \phi_{(c)}>}{<\phi^* F_{(c)} \phi_{(c)}>} \tag{3}$$

where the last term on the r.h.s. accounts for direct effects of the perturbation, while $\psi^*_{c(c)}$ and ψ_c obey equations

$$B^*_{(c)} \psi^*_{c(c)} + G^*_c \phi^*_c = 0 \tag{4}$$

and

$$B \psi_c + G_c \phi_{(c)} = 0 \tag{5}$$

where $B^*_{(c)}$ represents operator B^* at the state altered by the change $\delta_c B$ while

$$G_c = \delta_c B - \rho_c F_{(c)} \equiv \delta_c B_{(c)} - \rho_c F \tag{6}$$

Functions $\psi^*_{c(c)}$ and ψ_c are assumed void of the fundamental modes $\phi^*_{(c)}$ and ϕ, respectively. In order to assure this condition, available codes, as ERANOS, after calculating particular solutions $\psi^*_{(c),part}$ and ϕ_{part}, filter them along expressions:

$$\psi^*_{c(c)} = \psi^*_{c(c),part} - \frac{<\psi^*_{c(c),part} F_{(c)} \phi_{(c)}>}{<\phi^*_{(c)} F_{(c)} \phi_{(c)}>} \phi^*_{(c)} \tag{7}$$

$$\psi_c = \psi_{c,part} - \frac{\phi^* F \psi_{c,part}>}{<\phi^* F \phi>} \phi \tag{8}$$

Considering then the EGPT methodology, it is shown that functions $\psi^*_{c(c)}$ and ψ_c coincide with functions $\delta_c \phi^* \equiv (\phi^*_{(c)} - \phi^*)$ and $\delta_c \phi \equiv (\phi_{(c)} - \phi)$, obeying equations

$$B^*_{(c)} \delta_c \phi^* + G^*_c \phi^*_{(c)} = 0 \tag{9}$$

and

$$B \delta_c \phi + G_c \phi_{(c)} = 0 \tag{10}$$

[1] The GPT perturbation expression considered here represents an improvement over those normally encountered: the function $\psi^*_{c(c)}$ (governed by operator $B^*_{(c)}$) is used rather than ψ^*_c (governed by operator B^*).

Since functions $\delta_c \phi^*$ and $\delta_c \phi$ must be also void of the fundamental modes $\phi^*_{(c)}$ and ϕ, they are replaced, respectively, by:

$$-\phi^* - \frac{<\phi^* F_{(c)} \phi_{(c)}>}{<\phi^*_{(c)} F_{(c)} \phi_{(c)}>} \phi^*_{(c)} \tag{11}$$

and

$$\phi_{(c)} - \frac{<\phi^* F \phi_{(c)}>}{<\phi^* F \phi>} \phi \tag{12}$$

We may then write

$$\delta Q = \frac{<\phi^*_{(c)} \delta_s B_{(c)} \phi_{(c)}>}{<\phi^*_{(c)} F_{(c)} \phi_{(c)}>} - \frac{<\phi^* \delta_s B_{(c)} \phi_{(c)}>}{<\phi^* F_{(c)} \phi_{(c)}>} - \frac{<\phi^* \delta_s B \phi>}{<\phi^* F \phi>} + \frac{<\phi^* \delta_s B \phi_{(c)}>}{<\phi^* F_{(c)} \phi_{(c)}>} - \rho_c \frac{<\phi^* \delta_s F \phi_{(c)}>}{<\phi^* F_{(c)} \phi_{(c)}>} \tag{13}$$

Since $\delta_c B_{(c)} = \delta_c B - \rho_c \delta_s F$, expression (13) may be simplified and obtain the EGPT expression:

$$(\delta Q)_{EGPT} = \frac{<\phi^*_{(c)} \delta_s B_{(c)} \phi_{(c)}>}{<\phi^*_{(c)} F_{(c)} \phi_{(c)}>} - \frac{<\phi^* \delta_s B \phi>}{<\phi^* F \phi>} \tag{14}$$

The r.h.s, of this equation could be interpreted as the difference of the first-order reactivity changes induced by $\delta_s B$ in modified (by $\delta_c B$) and unmodified conditions. In terms of multiplication coefficient (k) we can write:

$$\delta_s \rho_c = \left(\frac{1}{k_{(s)}} - \frac{1}{k_{(cs)}}\right) - \left(\frac{1}{k} - \frac{1}{k_{(c)}}\right) \equiv \left(\frac{1}{k_{(c)}} - \frac{1}{k_{(cs)}}\right) - \left(\frac{1}{k} - \frac{1}{k_{(s)}}\right) \tag{15}$$

where:

- $k = k_{eff}$ value at reference condition
- $k_{(c)} = k_{eff}$ value after the perturbation considered in functional Q at reference conditions
- $k_{(s)} = k_{eff}$ value after the system modification considered
- $k_{(cs)} = k_{eff}$ value after the perturbation considered in functional Q at system modified conditions.

The first and second term at right hand side of Eqs. (14) and (15) correspond, respectively, to the perturbation reactivity effect $(1/k_{(c)} - 1/k_{(cs)})$ $[\equiv \rho_{(cs)} - \rho_{(c)}]$ on the modified system and to the perturbation reactivity effect $(1/k - 1/k_{(s)})$ $[\equiv \rho_{(s)} - \rho]$ on the unmodified one.

3. Deterministic and Monte Carlo calculation methods

The EGPT methodology described above implies the calculation of reactivity worths. In the exercise which we shall illustrate in Section 5 the MCNP 6.1 (Denise Pelowitz, 2012) and ERANOS (Rimpault et al., 2002) codes are considered for this purpose. More specifically, the adjoint (GPT) modality of ERANOS and the forward adjoint weighted tally of MCNP 6.1 have been used for comparison and verification of the results.

3.1. The Monte Carlo code in the adjoint field modality

Since 1994, MCNP 4C was able to perform simulations in adjoint modality by using a multi-group set of cross section data (Wagner et al., 1994). The so called multi-group/adjoint capability has been mainly developed for code comparisons and to enhance the calculation efficiency. The MCNP 4C adjoint option can only be used in the multi-group mode without possibility of performing adjoint