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Stochastic processes during transients in nuclear reactors

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ABSTRACT

Steady state noise techniques are widely known and applied to the monitoring of neutron reacting system. This paper deals with the stochastic analysis of neutron chain systems (nuclear reactors or fissile system) which are changing in time from subcritical states reaching other subcritical, critical or hypercritical states due to an external parametric excitation. Two cases are analyzed: 1) without reactivity feedback, that is from a subcritical state to one with almost zero power, and 2) a supercritical excursion with thermalhydraulic feedback. Our goal is to check in case 1 if the usual noise techniques can be used to monitoring the reactivity changes and in case 2 how to calculate the variance of the power and energy released.

1. Introduction

Due to the random nature of nuclear processes (fission in particular), nuclear reactors have a fluctuating neutron population which is more evident at very low power levels. For steady subcritical and critical cases the theory of neutron noise is very well-known and many noise techniques were developed to measure nuclear parameters under steady conditions (Williams, 1974). Under transient conditions, when the reactor parameters are changing either under control or not, at least two questions arise: 1) if the system is under control, how the parameters that define the reactivity have to change in order to have a quasi-equilibrium value for the noise signatures during the transient, in other words under which conditions we can apply known steady noise techniques to monitor the transient, 2) for the case of an accidental reactivity transient how to compute the fluctuations of the power pulse and the energy release. We answer both questions with the analysis of two real examples: for example, the monitoring of the manipulations of fissile solutions (Mihalczo et al., 1990) and the reactor accident analyzed by Difilippo (2015). The next section summarizes the model and the mathematical tools for the analysis of the non steady stochastic processes.

2. Probabilistic distribution of neutrons and the integrated number of fissions for non-Steady state

In this section we generate a set of differential equations for the first two moments of the distribution of the number of neutrons n at time t and the integral of the numbers of fissions f, up to time t, for a reacting system changing in time. We use the prompt one-point kinetics to compute the time dependent fluctuations. Because we want to analyze the case of parametric excitation our point reactor parameters are now explicit function of time t (due to the controlled, or not, parametric external changes) and function of the state variables T (for short) due to thermalhydraulic feedbacks.

2.1. Probability distribution function

Defining $P(n, f, t)$ as the probability of having at time t, n neutrons in the system and f integrated number of fissions in our reactor a difference-differential equation, or probability balance equation, can be written for $P(n, f, t)$ provided we have processes defined in the following way: $S(t)\Delta t$ is the probability for the emission of one neutron by the source in the interval Δt around t, similarly $\Lambda_f(t,T)\Delta t$, $\Lambda_c(t,T)\Delta t$, are the probabilities per neutron to have, respectively, a fission or a capture at time t; note that this probabilities depends explicitly on time (due to the external parametric excitation) and the set of thermalhydraulic variables (densities, temperatures, pressures etc), called "T " for short, due to the reactivity feedback. $P(n, f, t+\Delta t)$ is related to $P(n, f, t)$ by the equation

$$P(n,f,t+\Delta t) = P(n-1,f,t)S(t)\Delta t + P(n+1,f,t)(n+1)\Lambda_c\Delta t + \sum_{j=1}^{\infty} P(n-j+1,f-1,t)\Lambda_f(n-j+1)p(j) + [1-(S(t)+\Lambda_c n+\Lambda_f n)\Delta t]P(n,f,t) \quad (1)$$

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Each term in the right hand side of this equation indicates the contributions to the state (*n,f*) at time $t+\Delta t$ from states a *t:* 1) from state (*n–1,f*) via a neutron source contribution; 2) from state (*n+1, f*) via a capture process; 3) from a state (*n–j+1,f–1*) via an absorption process that produces a fission and *j* prompt neutrons with probability *p*(*j*); and 4) from a state (*n,f*) when nothing happens in Δt. Note we are making the hypothesis that times *t* are short compared with the decay times of the delayed neutron precursors, i.e. we are using the prompt approximation. By multiplying both sides by $x^n z^f$ and summing over all the possible values of *n* and *f* we obtain that the probability generating functions

$$\begin{aligned}
F(x,z,t) &= \sum_{n=0}^{\infty}\sum_{f=0}^{\infty} P(n,f,t)x^n z^f \\
F_c(x,z,t) &= \sum_{n=0}^{\infty}\sum_{f=0}^{\infty} \Lambda_c(t,T)P(n,f,t)x^n z^f \\
F_f(x,z,t) &= \sum_{n=0}^{\infty}\sum_{f=0}^{\infty} \Lambda_f(t,T)P(n,f,t)x^n z^f
\end{aligned} \tag{2}$$

satisfy the equation

$$\frac{\partial F}{\partial t} = S(t)(x-1)F + \frac{\partial F_c}{\partial x}(1-x) + [zp(x)-x]\frac{\partial F_f}{\partial x} \tag{3}$$

where we emphasize the changes in time of S, and the parametric dependence of the nuclear parameters Λ_f and Λ_c. For the case of constant reactor parameter Eq. (3) was analyzed in detail by Pacilio (1976) Eq. (3), $p(x)$ is

$$p(x) = \sum_{\nu_p=0}^{\infty} x^{\nu_p} p_{\nu_p} \tag{4.1}$$

where p_{ν_p} is the probability of the emission of ν_p prompt neutrons in the fission process, of course the "infinite" in Eqs. (1) and (4.1) are formal, in reality up to "what we know" of the fission process. The factorial moments of the number of prompt neutron appears in the next equation so for further use we define

$$\bar{\nu}_p = \left(\frac{\partial p(x)}{\partial x}\right)_{x=1} = \sum_{\nu_p=0}^{\infty} \nu_p p_{\nu_p} \tag{4.2}$$

$$< \nu_p(\nu_p-1) >= \left(\frac{\partial^2 p(x)}{\partial x^2}\right)_{x=1} = \sum_{\nu_p=0}^{\infty} \nu_p(\nu_p-1)p_{\nu_p} \tag{4.3}$$

Any moment of the distribution *P*(*n, f, t*) can be calculated with the factorial moments which are solutions of a system of ordinary differential equations. More explicitly, the $(m+i)$ partial derivative of *F* with respect to *x* (*m* times) and with respect to *z* (*i* times) evaluated with Eq. (2) at *x* = *z* = *1* is

$$\left(\frac{\partial^{(m+i)}}{\partial x^m \partial z^i}\frac{\partial F}{\partial t}\right)_{x=z=1} = \langle n(n-1)(n-2)\ldots(n-m+1)f(f-1)(f-2)\ldots(f-i+1)\rangle \tag{5}$$

where the bracket indicate the average with the distribution *P*(*n,f, t*).Now these equations are not sufficient to compute the moments because of the parametric excitations so we need the additional modeling of the external reactivity changes and the thermodynamic feedback.

2.2. System of differential equations for the factorial moments

The partial derivatives of $\partial F/\partial t$ in Eq. (5) are calculated according to the right hand side of Eq. (3). Because we cannot factorize Λ_c and Λ_f out of the sums in Eq. (2) there is a correlation between the reactor parameters and the neutron and fission distributions via the thermalhydraulic set of variables *T*. Explicitly for the first two moments we have

$$\frac{d<n>}{dt} = S(t) + <(\bar{\nu}_p\Lambda_f - \Lambda_a)n> \tag{6.1}$$

where $\Lambda_a = \Lambda_c + \Lambda_f$

$$\frac{d<f>}{dt} = <\Lambda_f n> \tag{6.2}$$

$$\frac{d<n(n-1)>}{dt} = 2S<n> + <\nu_p(\nu_p-1)><\Lambda_f n> + 2<(\bar{\nu}_p\Lambda_f - \Lambda_a)n(n-1)> \tag{7.1}$$

$$\frac{d<nf>}{dt} = <(\bar{\nu}_p\Lambda_f - \Lambda_a)nf> + S<f> + <\Lambda_f n(n-1)> + \bar{\nu}_p<\Lambda_f n> \tag{7.2}$$

and

$$\frac{d<f(f-1)>}{dt} = 2<\Lambda_f nf> \tag{7.3}$$

We introduce now the more standard parameters: mean life $l = 1/\Lambda_a$, multiplication constant $k = \bar{\nu}\Lambda_f/\Lambda_a$, the reactivity, $\rho = (k-1)/k$, the prompt decay constant $\alpha = -\Lambda_a + \bar{\nu}_p\Lambda_f = (\rho-\beta)/\Lambda_g$, where $\Lambda_g = l/k$ is the generation time. Eqs. (6) and (7) can be written as

$$\frac{d<n>}{dt} = S(t) + <\alpha n> \tag{8.1}$$

$$\frac{d<f>}{dt} = <n/\Lambda_g>/\bar{\nu} \tag{8.2}$$

$$\frac{d<n(n-1)>}{dt} = 2S<n> + <\nu_p(\nu_p-1)><n/\Lambda_g>/\bar{\nu} + 2<\alpha n(n-1)> \tag{9.1}$$

$$\frac{d<nf>}{dt} = <\alpha nf> + S<f> + <n(n-1)/\Lambda_g>/\bar{\nu} + (1-\beta)<n/\Lambda_g> \tag{9.2}$$

$$\frac{d<f(f-1)>}{dt} = 2<nf/\Lambda_g>/\bar{\nu} \tag{9.3}$$

Examples for the solutions of Eqs. (8) and (9) are given for two cases: the first one a subcritical transient with the reactivity changing in time and without any thermal feedback or dependence on *n*, for this case we have exact solutions; the second case corresponds to the accidental supercritical power excursion recently described by Difilippo (2015); because of the thermal hydraulic feedback (that depends on *n*) some simplifying hypothesis has to be included for this case.

3. Fluctuations during a transient between two subcritical states without thermal hydraulic feedback

Changes in the system are usually monitored with a neutron source and a detector, examples are the approach to the critical state of any reactor or, for example, the monitoring of changes in fissile solutions with noise techniques (Mihalczo et al., 1990). Because of the assumption of very low power no thermohydraulic effects exist and the changes of the nuclear parameter are only function of time and therefore can be factorized out of the averages in Eqs. (8) and (9),