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Technical note

Two-dimensional hexagonal geometry discontinuity factors at the core periphery

Nicholas P. Luciano, G. Ivan Maldonado *

The University of Tennessee, Department of Nuclear Engineering, 311 Pasqua Engineering Building, Knoxville, TN 37996-2300, United States

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ABSTRACT

The proper calculation of core periphery discontinuity factors is important for accurate modeling when using an advanced nodal diffusion simulator. In cores with hexagonal assemblies, such as in the VVER-1000, most fuel assemblies share two faces with the radial reflector, and some even three faces. For this reason, use of a two-dimensional (2D) reflector model will more accurately capture the neutron physics near the core periphery. This article illustrates key points related to the use of 2D discontinuity factors in the reflector region, first by using an algorithm that applies the methodology proposed by Mittag et al. (2003) after correcting some minor typographical errors in the original publication, and then by employing the SCALE transport module NEWT to compute the appropriate quantities. Large and even negative discontinuity factors are an acceptable fact of this methodology when the diffusion approximation becomes invalid due to the problem's localized features and the large flux gradients.

1. Introduction

In western-style LWRs with square pitched assemblies, most peripheral fuel assemblies share one face with the radial reflector region, and only a few corner assemblies share two boundaries. For these cores, a one-dimensional transport-based reflector model can be used to generate homogenized few-group constants for the core/reflector interface and obtain assembly powers with a nodal diffusion simulator that are reasonably accurate in most cases (Tahara et al., 2000; Müller, 1989). In many cores with hexagonal assemblies the situation is different because only a few hexagonal reflector nodes share a single face with peripheral fuel nodes. In fact, most share two faces, and some even share three faces. Because of this feature of the geometry, a one-dimensional reflector model is less accurate for cores with hexagonal assemblies. In general, using a two-dimensional reflector model will have a greater benefit upon assembly power accuracy near the periphery for a VVER-1000 core that it would for a western-style LWR core.

Lattice physics codes, such as the NEWT module within the SCALE code suite (DeHart and Bowman, 2011), can calculate the homogenized few-group macroscopic cross sections for a selected region of the model. In addition, a nodal core simulator such as the NESTLE code requires group discontinuity factors on each node's face. SCALE and the NESTLE nodal core simulator have been used to accurately model a VVER-1000 operational benchmark (Luciano et al., 2016). Computing the discontinuity factors for the zero net neutron current case is straightforward. It is more difficult to compute discontinuity factors with net currents on the node faces, such as in a reflector region. While NEWT can compute discontinuity factors, it does not do so when a non-zero net current is present on the node faces. To calculate its overall flux solution, NEWT computes the group fluxes and partial currents along many grid lines in the geometry. By using the *fluxplane* feature of the code, group fluxes and currents for any line coincident with the grid lines are written to the output file. This feature is useful for calculating two-group discontinuity factors after the NEWT computation is complete.

2. Two-group discontinuity factors in the radial reflector

A method to compute discontinuity factors for non-multiplying material in two-dimensional hexagonal reactor geometry has been presented by Mittag et al. (2003). The method is based on expanding the group fluxes using:

$$\phi_g(x,y) = \sum_{n=1}^{N} C_{g,n}^{+} e^{\mu_g(k_{n,x}x + k_{n,y}y)} + C_{g,n}^{-} e^{-\mu_g(k_{n,x}x + k_{n,y}y)} \quad (1)$$

where $N = 3, k_{n,x} = \cos\theta_n, k_{n,y} = \sin\theta_n, \theta_n = \pi(n-1)/N$. The constants μ_g are based on the homogenized group cross sections, and $C_{g,n}^{\pm}$ are coefficients to be determined. The method is a two-group

* Corresponding author.
E-mail address: ivan.maldonado@utk.edu (G.I. Maldonado).

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formulation and would require significant modifications for groups other than two. A derivation of the method is presented in the reference, and is not repeated here. Instead the method is presented here in algorithmic form, where some minor typographic errors from the original publication have been corrected as illustrated within the box below:

In the original publication, Eq. (18) was written:

$$J_{1,1}^{\mu,1} = -D_1\mu\left(C_1^{\mu+}e^{\mu d/2} - C_1^{\mu-}e^{-\mu d/2}\right)$$

but should be corrected to:

$$J_{1,1}^{\mu,1} = \frac{d\mu}{\sqrt{3}}\left(C_1^{\mu+}e^{\mu d/2} - C_1^{\mu-}e^{-\mu d/2}\right) \tag{2}$$

In the original publication, Eq. (24) was written:

$$\phi_2^{\text{hom},s} = \alpha\phi_1^{\text{hom},s} + \frac{\sqrt{3}}{d\mu}\sum_{n=1}^{3}\left\{\frac{C_n^{v+}}{k_{n,y}^s}\left[e^{(vd/2)(k_{n,x}^s+k_{n,y}^s/\sqrt{3})}e^{(vd/2)(k_{n,x}^s-k_{n,y}^s/\sqrt{3})}\right] - \frac{C_n^{v-}}{k_{n,y}^s}\left[e^{(-vd/2)(k_{n,x}^s+k_{n,y}^s/\sqrt{3})}e^{(-vd/2)(k_{n,x}^s-k_{n,y}^s/\sqrt{3})}\right]\right\}$$

but should be corrected to:

$$\phi_2^{\text{hom},s} = \alpha\phi_1^{\text{hom},s} + \frac{\sqrt{3}}{dv}\sum_{n=1}^{3}\left\{\frac{C_n^{v+}}{k_{n,y}^s}\left[e^{(vd/2)(k_{n,x}^s+k_{n,y}^s/\sqrt{3})}e^{(vd/2)(k_{n,x}^s-k_{n,y}^s/\sqrt{3})}\right] - \frac{C_n^{v-}}{k_{n,y}^s}\left[e^{(-vd/2)(k_{n,x}^s+k_{n,y}^s/\sqrt{3})}e^{(-vd/2)(k_{n,x}^s-k_{n,y}^s/\sqrt{3})}\right]\right\} \tag{3}$$

2.1. Algorithm

1. From the transport solution, obtain the net currents $J_{g,s}^{\text{Net}}$ and the heterogeneous fluxes $\phi_{g,s}^{\text{Het}}$ along each side of the hexagon. The net currents are defined to be positive when the current direction is outward from the hexagon, and negative when the current direction is inward. Currents reported by NEWT are positive in the $+x$ and $+y$ directions. Therefore, the following equation must be used:

$$J_{g,s}^{\text{Net}} = J_{g,s}^{x+}\cos\theta_s + J_{g,s}^{y+}\sin\theta_s \tag{4}$$

where θ_s defines the outward normal for each side; in this case $\theta_s = \left[0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}\right]$ for sides = [East, Northeast, Northwest, West, Southwest, Southeast].

2. The two-group homogenized cross sections are used to define three parameters:

$$\mu_1 = \sqrt{\Sigma_{\text{R},1}/D_1} \tag{5}$$

$$\mu_2 = \sqrt{\Sigma_{\text{a},2}/D_2} \tag{6}$$

$$\alpha = \frac{\Sigma_{\text{s},1\to2}}{\Sigma_{\text{a},2} - D_2\mu_1^2} \tag{7}$$

where Σ_{a} is the group absorption macroscopic cross section, Σ_{s} is the group to group scattering macroscopic cross section, D is the diffusion coefficient, and $\Sigma_{\text{R},1} = \Sigma_{\text{a},1} + \Sigma_{\text{s},1\to2}$ is the fast group removal macroscopic cross section.

3. Compute the 6×6 matrices $\mathbf{M}_g$:

$$\mathbf{M}_g = \begin{bmatrix} m_{g,0}^+ & m_{g,0}^- & M_{g,1,2}^+ & M_{g,1,2}^- & M_{g,1,3}^+ & M_{g,1,3}^- \\ M_{g,2,1}^+ & M_{g,2,1}^- & m_{g,0}^+ & m_{g,0}^- & M_{g,2,3}^+ & M_{g,2,3}^- \\ M_{g,3,1}^+ & M_{g,3,1}^- & M_{g,3,2}^+ & M_{g,3,2}^- & m_{g,0}^+ & m_{g,0}^- \\ m_{g,0}^- & m_{g,0}^+ & M_{g,4,2}^+ & M_{g,4,2}^- & M_{g,4,3}^+ & M_{g,4,3}^- \\ M_{g,5,1}^+ & M_{g,5,1}^- & m_{g,0}^- & m_{g,0}^+ & M_{g,5,3}^+ & M_{g,5,3}^- \\ M_{g,6,1}^+ & M_{g,6,1}^- & M_{g,6,2}^+ & M_{g,6,2}^- & m_{g,0}^- & m_{g,0}^+ \end{bmatrix} \tag{8}$$

where

$$m_{g,0}^{\pm} = \pm\mu e^{\pm\mu_g d/2} \tag{9}$$

$$M_{g,s,n}^{\pm} = \frac{k_{n,x}^s}{k_{n,y}^s}\left(e^{\pm\mu_g d/2(k_{n,x}^s+k_{n,y}^s/\sqrt{3})}) - e^{\pm\mu_g d/2(k_{n,x}^s-k_{n,y}^s/\sqrt{3})}\right) \tag{10}$$

with

$$k_{n,x}^s = \cos(\theta_n - \theta_s) \tag{11}$$

$$k_{n,y}^s = \sin(\theta_n - \theta_s) \tag{12}$$

$$\theta_s = \frac{2\pi(s-1)}{6} \tag{13}$$

$$\theta_n = \frac{\pi(n-1)}{3} \tag{14}$$

and d is the distance between parallel sides of the regular hexagon (equivalent to the pitch of a regular hexagonal lattice).

4. For the fast and thermal groups, solve the matrix equations:

$$\vec{J_1^{\text{Net}}} = -\frac{D_1\sqrt{3}}{d}\mathbf{M}_1\vec{C_1} \tag{15}$$

$$\vec{J_2^{\text{Net}}} = -\frac{D_2\sqrt{3}}{d}\mathbf{M}_2\vec{C_2} + \frac{\alpha D_2}{D_1}\vec{J_1^{\text{Net}}} \tag{16}$$

for the 1×6 group coefficient vectors $\vec{C_g}$

5. Using the coefficient vectors $\vec{C_g}$, solve the matrix equations:

$$\vec{\phi_1^{\text{Hom}}} = \frac{\sqrt{3}}{\mu_g d}\mathbf{F}_1\vec{C_1} \tag{17}$$

$$\vec{\phi_2^{\text{Hom}}} = \frac{\sqrt{3}}{\mu_g d}\mathbf{F}_2\vec{C_2} + \alpha\vec{\phi_1^{\text{Hom}}} \tag{18}$$

for the fast and thermal homogeneous fluxes $\vec{\phi_g^{\text{Hom}}}$, where the 6×6 matrix:

$$\mathbf{F}_g = \begin{bmatrix} f_{g,0}^+ & f_{g,0}^- & F_{g,1,2}^+ & F_{g,1,2}^- & F_{g,1,3}^+ & F_{g,1,3}^- \\ F_{g,2,1}^+ & F_{g,2,1}^- & f_{g,0}^+ & f_{g,0}^- & F_{g,2,3}^+ & F_{g,2,3}^- \\ F_{g,3,1}^+ & F_{g,3,1}^- & F_{g,3,2}^+ & F_{g,3,2}^- & f_{g,0}^+ & f_{g,0}^- \\ f_{g,0}^- & f_{g,0}^+ & F_{g,4,2}^+ & F_{g,4,2}^- & F_{g,4,3}^+ & F_{g,4,3}^- \\ F_{g,5,1}^+ & F_{g,5,1}^- & f_{g,0}^- & f_{g,0}^+ & F_{g,5,3}^+ & F_{g,5,3}^- \\ F_{g,6,1}^+ & F_{g,6,1}^- & F_{g,6,2}^+ & F_{g,6,2}^- & f_{g,0}^- & f_{g,0}^+ \end{bmatrix} \tag{19}$$

with

$$f_{g,0}^{\pm} = \frac{\mu_g d}{\sqrt{3}}e^{\pm\mu_g d/2} \tag{20}$$

$$F_{g,s,n}^{\pm} = \frac{\pm 1}{k_{n,y}^s}\left(e^{\pm\mu_g d/2(k_{n,x}^s+k_{n,y}^s/\sqrt{3})}) - e^{\pm\mu_g d/2(k_{n,x}^s-k_{n,y}^s/\sqrt{3})}\right) \tag{21}$$