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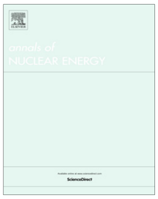

Natural frequencies of a CANDU fuel string

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ABSTRACT

A CANDU fuel string consists of 12 or 13 fuel bundles that laid horizontally inside a pressure tube. Under the large amount of drag load induced by the coolant flow the behaviour of individual bundles are coupled together through friction and contact. Presence of large number of contact and friction constraints causes the dynamical behaviour of many mechanical systems, like the vibration of CANDU fuel string, to be very complex. In this paper a numerical method based on linear complementarity problem (LCP) is developed to simulate vibrational behaviour of such systems. Then the presented method is employed and for the first time, natural frequencies of a CANDU fuel string are obtained numerically. Knowing the natural frequency of the string is very beneficial and can help to mitigate and decrease the damages, improve new fuel designs and develop new safety standards. All the solid components are discretized in space domain by the means of finite element method. Bozzak-Newmark integration scheme is employed to discretize the system equations of motion in the time-domain. With the computational power available today, frictional contact among fuel elements via spacer pads, between fuel elements and the pressure tube via bearing pads, and between neighbouring fuel bundles via endplates are modelled and the response of the string is obtained. Two different fuel string consist of 12 and 13 bundles are studied in this paper. FFT analyses are performed and natural frequencies of the systems are extracted. Results show great agreement with experimental values. The effect of boundary conditions in the last endplate of the downstream bundle on the natural frequencies is also investigated.

1. Introduction

Inside a CANDU reactor fuel channel, a string of fuel bundles (typically 12 or 13) is placed horizontally as it is illustrated in Fig. 1. A CANDU reactor core consists of a few hundred horizontally laid fuel channels. Heavy water coolant enters into the pressure tube at the inlet, passes through all fuel bundles and brings out the heat generated by each fuel element. The coolant that flows inside the pressure tube is highly turbulent and produces unsteady drag, side, and lift forces that cause bundles to vibrate during operation (Norsworthy and Ditschun, 1995). The flow-induced drag pushes all the bundles toward the shield plug or side stoppers at the outlet of the fuel channel and forms a coherent structure consist of several bundles. The behaviour of individual fuel bundles is coupled together through contact and friction. The vibration of a CANDU fuel string inside a pressure tube is an extremely challenging problem from many aspects. One of the main challenges comes from the unilateral frictional contact constraints between the fuel elements and pressure tube, between the neighbouring fuel elements and between neighbouring bundles. To quantify vibration of a string of fuel bundles in a fuel channel, dynamic contact constraints between all potential contact surfaces must be accounted for. UFC is a very challenging problem especially for a large-scale system like the CANDU fuel string. The non-smooth body-to-body interactions vary in both spatial and time domains and direction and magnitude of friction and contact forces are not known a priori.

Understanding the dynamical behaviour of the fuel string and knowing its natural frequencies is very crucial and would result in decreasing the undesired damages and improving the new fuel designs. It may also help us to improve existing safety standards and introduce new safety codes. One example would be the acoustic pressure pulsation problem in Darlington reactor. Lau et al. (1992) reported that the main pumps vane passing frequency of 150 Hz is very close to one of the natural frequencies of the fuel bundle string. The investigations on the endplate failure in the Darlington reactor led to the replacement of the 5-vane pump with a 7-vane pump in order to attenuate the acoustic-structure resonance associated with the fundamental blade passing frequency (moving the pressure pulsation frequency from 150 Hz to 210 Hz). It could have saved lots of money and time if it was

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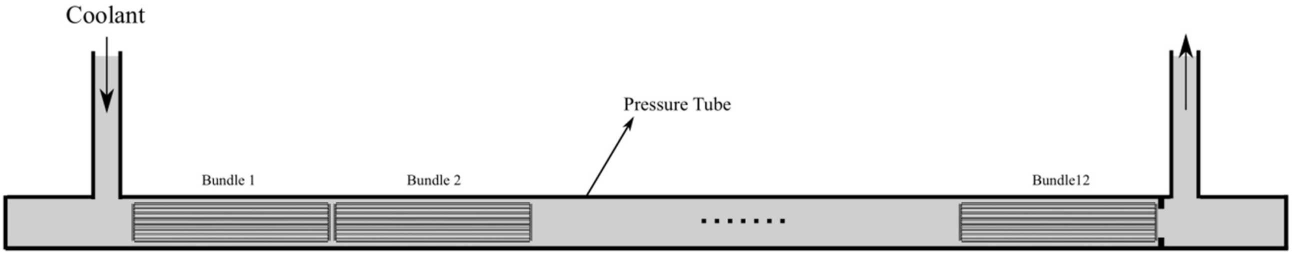

Fig. 1. A CANDU fuel bundle string consisting of n bundles.

known that one of the natural frequencies of the string is very close to 150 Hz. Natural frequencies of the fuel string can be found using numerical method or by performing experimental studies. Lau et al. (1992) have reported the experimentally found frequencies of the peak response for the endplates displacements. The tests were performed with a 13-bundle fuel string at a temperature of 59 C and with a mass flow rate of 31.4 kg/s. Lau et al. (1992) have concluded that most of these frequencies likely correspond to the natural frequencies of the fuel string, however some of those frequencies may be due to the presence of fluid flow and the acoustic affects. Conducting experimental studies is very expensive, time consuming and needed to be performed for any new setup and design.

The other alternative for obtaining natural frequencies is the numerical methods. A comprehensive and efficient dynamic model consists of all the components, i.e. fuel elements, endplates etc., and interaction between them is necessary in order to truly investigate the dynamic behaviour of the fuel string. As is pointed out by Hassan and Rogers (Hassan and Rogers, 2005), if a contact procedure/algorithm requires exhaustive search and iterations to achieve a solution of acceptable accuracy at one time step, such an approach may not be feasible in a problem with large number of contact constraints like CANDU fuel string.

Literature review reveals that up to date there is no such a comprehensive dynamic model of the fuel string with the consideration of all 2D friction and contact constraints. In this study, a numerical method by the means of LCP and Bozzak-Newmark time-stepping scheme is developed and employed to obtain a vibrational response of the fuel string by considering all the frictional contact constraints. Two-dimensional friction between different components of the fuel string is handled as it was explained by Fadaee and Yu (2015). Then through the introduction of a novel variable transformation, the response of the dynamical system subjected to multiple non-smooth constraints at a time step is reduced to a quadratic programming problem, or a LCP for which an accurate solution may be obtained efficiently using the Lemke algorithm. The main advantage of the LCP is that at every time step, the solution satisfies all the contact and frictional constraints simultaneously without iterations.

2. Description of the dynamical system

During operation, under the flow-induced drag individual fuel bundles form a fuel string inside a pressure tube, like the one shown in Fig. 1 that consists of 12 fuel bundles. Because of the gravity and radial clearances, the fuel string tends to rest on the bottom of the pressure tube near the 6 o'clock position through bearing pads. Coupled with large-scale contact constraints between different components, the so-formed fuel string can exhibit very complex dynamic behaviours under fluid flow excitations. Fuel rods are the most important constitutive components of a fuel bundle. A CANDU fuel rod contains a number of short-dished UO_2 pellets inside a thin hollow Zircaloy sheath sealed at the ends with two endcaps. The pellets and sheath form an integral compound beam-like structure with a large length-to-diameter ratio (about 40). In this paper fuel rods are modelled as composite beams (Tayal, 1989) with different material properties for UO_2 and Zircaloy. It is assumed that all the radial and axial gaps between the pellets and sheath are zero and pellets are not free to slide over the inner surface of the sheath.

A 37-element fuel bundle has four concentric rings of fuel elements (one at the centre, 6 at the inner ring, 12 at the intermediate ring, and 18 at the outer ring, see Fig. 2). All fuel elements in a bundle are spaced radially and circumferentially to allow the passage of coolant over their outside surfaces. As is shows in Fig. 2, the radial and circumferential gaps between neighbouring fuel elements are designed and maintained by means of spacer pads for eliminating the large-scale element-to-element contact that creates localized hot spots. Bearing pads are introduced to the outer ring fuel elements at three or more bundle cross-sections to prevent large area contact between fuel elements and pressure tube (see Figs. 2 and 3). A 37-element endplate, made of Zircaloy, consists of three concentric circular rings to hold together 36 non-centre fuel elements at their designed positions. As it is illustrated in Fig. 3 circumferentially spaced radial ribs are used to connect the inner, intermediate and outer endplate rings and hold them at their designated radial positions. A pair of straight webs connected to the inner ring secure the positions of the 37th fuel element at the centre. In this study all solid components are discretized in the space domain by means of the finite element method. A three-node higher-order mixed beam finite element (Yu and Fadaee, 2012) is used for modelling of fuel rods as straight compound beams for coupled bending-axial and bending-torsional vibration. The special nine-node thick plate finite elements (Yu and Fadaee, 2016) is employed for the in-plane and out-of-plane deformations of endplates. The finite element discretization of solid components yields a large number of DOF's for a fuel string. Since only a small percentage of the total DOF's is involved in the UFC formulations, the component substructure method (Bathe, 1997) is used to eliminate the interior DOF's and significantly reduce the dimensions of the non-smooth dynamical problem.

Equation of motion for a single fuel rod may be written as

$$[m]\{\ddot{q}\} + [c]\{\dot{q}\} + [k]\{q\} = \{Q\}_{i+1} + \{Q_f\} + \{Q_c\} + \{R_{ep-fe}\} \tag{1}$$

where $[m]$, $[k]$ and $[c]$ are the mass, stiffness and damping matrices and are obtained using finite element method as it explained by Yu and Fadaee (2012); $\{Q_c\}$ and $\{Q_f\}$ are the generalized force vectors associated with contact and frictional forces; $\{Q\}$ is the excitation force vector; $\{R_{ep-fe}\}$ is the restraining force vector acting on fuel elements from left and right endplates due to the continuity. The reaction forces acting on endplates are $\{R_{fe-ep}\}$ (Note that $\{R_{ep-fe}\} = -\{R_{fe-ep}\}$).

Assume that the state of the vibrational system at $t = t_i$ is determined. The state of the system at $t = t_{i+1}$ may be found by solving the following equations (Rao, 2010)

$$\begin{aligned}(1+\alpha)[m]\{\ddot{q}\}_{i+1} - \alpha[m]\{\ddot{q}\}_{i+1} + [c]\{\dot{q}\}_{i+1} + [k]\{q\}_{i+1} \\ = \{Q\}_{i+1} + \{Q_f\}_{i+1} + \{Q_c\}_{i+1} + \{R_{ep-fe}\}_{i+1}\end{aligned} \tag{2}$$