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Application of Extreme Learning Machines to inverse neutron kinetics

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ABSTRACT

The paper presents the application of Extreme Leaning Machines (ELMs) for inverse reactor kinetic applications. ELMs were proposed by Huang and co-workers (2004, 2006a,b, 2015), which showed their enhances capabilities in terms of training speed and generalization with respect to classical Artificial Neural Networks (ANNs). ELMs are here implemented for reactivity determination as an alternative to ANNs (e.g. Picca et al. (2008)) and Gaussian Processes (Picca and Furfaro, 2012). After a review of the main features of ELMs, their application to inverse kinetic problems is proposed. The ELMs performance is tested on a typical accelerator drive system configuration (Yalina reactor) and the inversion is carried out on an accurate kinetic model (multi-group transport).

1. Introduction

The interpretation of experiments in accelerator-driven systems (ADS, OECD/NEA 2002) has recently offered new challenges for inverse reactor kinetics techniques. In fact, these subcritical configurations are characterized by compact cores with an inner highly-enriched booster surrounded by the region designed for nuclear waste transmutation (e.g. Salvatores et al. (1996), Soule et al. (2004), Kiyavitskaya et al. (2007)). The macroscopic heterogeneity of the typical ADS configurations introduces some new features in their dynamic behavior, which often cannot be accurately described with a standard lumped parameter model, such as the point kinetics (PK, Henry, 1958). For this reason, the classical kinetic inversion techniques based on the point kinetics shows inherent limitations when used to interpret experiments in these subcritical configurations (e.g. Eriksonn et al. (2005)).

In the last decade, attempts to overcome these limitations were made by considering different inversion approaches, e.g. based on Artificial Neural Networks (ANNs, Picca et al., 2008, 2009, 2010) or Gaussian Processes (GPs, Picca and Furfaro, 2012). The main advantage of these techniques is that they enable the inversion of more accurate reactor kinetics models, thus reducing the *modeling error* associated with the subcriticality estimation. As a drawback, these inversion approaches introduce an *inversion error*, which is typically not affecting other classical analytically-based techniques (e.g. Sjöstrand (1956)). The potential dependence of the inversion results on the network architecture represents a key aspect that needs to be validated before the practical application of the ANNs (e.g. see discussions in Picca et al. (2008, 2009, 2010)).

One of the great potential of neural-based inversion strategy is in its greater flexibility, which for instance allows the subcriticality level interpretation from the system response to multiple transients or on the interpretation of signals from several local detectors instead of the global thermal power (often not directly available in experiments). The main reason for a limited application of ANNs to more complex inversions (to Author's knowledge only in approached in Picca et al. (2010)) is associated with the computational burden of the backpropagation-based training of the ANNs for large training sets.

Huang and co-workers (2004, 2006a,b, 2015) developed the theory for the so-called extreme learning machines (ELMs) which effectively addresses the two main limitations (i.e., dependence of results on network architecture and computational effort for training). ELMs can be interpreted as a single hidden-layer neural network where the bias and the weights for the neuron in the inner layer are randomly generated. In Huang et al. (2006a,b), it was rigorously demonstrated that, under rather general conditions, ELMs can approximate any non-linear piecewise functions simply by training the coefficients in the output layer, i.e. without the need for iterative tuning typical of the optimization algorithms used in the backpropagation process. For this reason, the training operation for the ELMs can be carried out by one-pass least-square algorithm. Additionally, as demonstrated in a series of works (see the list at http://www.ntu.edu.sg/home/egbhuang/reference.html), this algorithm offers very interesting generalization performance with a remarkable independence of the results on the number of neurons in the inner layer.

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The motivation of the paper is twofold. Firstly, it extends the ANN-based inversion concept proposed in Picca et al. (2008, 2009, 2010)) implementing the powerful ELMs. Secondly, it considers a more realistic ADS configuration (Yalina-like reactor) and interprets its dynamic behavior on an advanced kinetic model such as the multi-group transport model, not attempted so far in literature to Author's knowledge.

The remainder of the paper is organized as follows. Section 2 presents the basic features of the inverse reactor kinetic problem and briefly reviews some classical inverse methods derived for PK model. Section 3 described the neural-based inversion approach for reactor kinetics, presenting both the standard ANNs and ELMs approaches. Results are provided in Section 4 before drawing some conclusions in Section 5.

2. Inverse problems in reactor kinetics

The time-dependent linear transport can be written in a multi-group approximation as follows (Akcasu et al., 1971):

$$\begin{cases} \left[\frac{1}{v}\frac{\partial}{\partial t} + \vec{\Omega}\cdot\nabla + \Sigma(\vec{x})\right]\varphi_g = \frac{1}{4\pi}\sum_{g'=1}^{G}\Sigma_{s,g'\to g}(\vec{x})\Phi_{g'}(\vec{x}) \\ \quad + \frac{(1-\sum_{i=1}^{R}\beta_i)\chi_g}{4\pi}\sum_{g'=1}^{G}\upsilon\Sigma_{f,g'}(\vec{x})\Phi_{g'}(\vec{x}) + \frac{\chi_{d,g}}{4\pi}\sum_{i=1}^{R}\lambda_i C_i(\vec{x},t) + \frac{q_g(\vec{x},t)}{4\pi} \\ \frac{\partial}{\partial t}C_i(\vec{x},t) = -\lambda_i C_i(\vec{x},t) + \beta_i\sum_{g'=1}^{G}\upsilon\Sigma_{f,g'}(\vec{x})\Phi_{g'}(\vec{x}) \end{cases} \quad (1)$$

where the scalar flux is defined as:

$$\Phi(\vec{x}) = \oint_{4\pi} d\vec{\Omega}'\varphi_g(\vec{x},\vec{\Omega}') \quad (2)$$

and standard definitions for the physical properties applies (see for example the definitions in Akcasu et al. (1971)).

A large class of inversion techniques developed in the past is based on the point approximation of the reactor kinetics. The PK is a lumped parameter model derived projecting Eq. (1) onto a weight function and hence condensing the spatial, energy and angular physical information in the coefficients of the system of first-order ODEs, i.e.:

$$\begin{cases} \frac{d}{dt}P(t) = \frac{\rho_{eff} - \sum_{i=1}^{R}\beta_{eff,i}}{\Lambda_{eff}}P(t) + \sum_{i=1}^{R}\lambda_i C_i(t) + S_{eff}(t) \\ \frac{d}{dt}C_i(t) = -\lambda_i C_i(t) + \beta_{eff,i}P(t) \end{cases} \quad (3)$$

where ρ_{eff}, $\beta_{eff,i}$ and Λ_{eff} are known as the kinetic parameters.

Among the methods based on PK, there are the area method (Sjöstrand, 1956), the slope fitting method (Simmons and King, 1958) and statistical methods based on the noise theory (Williams, 1974; Pázsit and Demazière, 2010). A large number of other classical methods for the inverse kinetics are available in literature (e.g. Feynmann et al. (1956), Orndoff (1957), Gozani (1962)). The choice of PK as the basis for subcriticality determination is particular convenient due to the simplicity of the point kinetic model as opposed to the initial integral-differential system of equations. For this reason, in many cases the inversion can be carried out analytically, thus avoiding *inversion error*. On the contrary, a major limitation of these inverse techniques is in the *model error* associated with the PK approximation. Although it si well know that the PK approximation is not very significant for large systems with no large macroscopic heterogeneity (e.g. Akcasu et al. (1971)), it can become particularly significant when considering ADSs (e.g., Eriksonn et al. (2005)). As an example, the Yalina configuration (Minsk, Belarus) comprises an inner highly enriched booster characterized by a fast neutron spectrum and an outer region with thermal spectrum (Kiyavitskaya et al., 2007). The two regions are separated by a layer which selectively absorbs thermal neutron to avoid a propagation of a transient in the periphery towards the centre of the system. Fig. 1 presents the layout of Yalina booster and Table 1 reports the material properties. For this configuration, the assumption of a point behavior of the system in pulsed transients cannot be easily justified since the shape variation during the transient is not negligible.

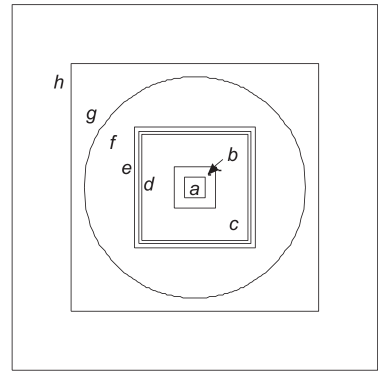

Fig. 1. Yalina booster core layout, 902EK configuration (Kiyavitskaya et al., 2007).

Table 1
Characterization of the material property of Yalina core, 902EK configuration (Kiyavitskaya et al., 2007).

Region	Mixtures	Geometry	% vol
Source (a)	Lead	Homog. block	0.292
Fast 1 (b)	Fuel: metal U 90% clad: SS 12X18H10T matrix: lead	Lattice cell Pitch = 1.143 cm	0.787
Fast 2 (c)	Fuel: metal U 36% clad: SS 12X18H10T matrix: lead	Lattice cell Pitch = 1.6 cm	6.885
Valve 1 (d)	Fuel: metal U 0.715% clad: SS 12X18H10T matrix: lead	Lattice cell Pitch = 1.6 cm	1.274
Valve 2 (e)	Absorber: B_4C clad: SS 12X18H10T matrix: lead	Lattice cell Pitch = 1.6 cm	1.367
Thermal (f)	Fuel: metal UO_2 + Mg 10% clad: Al alloy matrix: Polyethylene	Lattice cell Pitch = 2.0 cm	18.350
Poly (g)	Inner void holeclad: Al alloy matrix: Polyethylene	Lattice cell Pitch = 2.0 cm	14.802
Reflector (h)	Graphite	Homog. block	56.243

3. Neural-based inversion of the reactor kinetics

In this section, the neural-based inversion strategy for the reactor kinetic equations is reviewed, highlighting how it can overcome some of the limitations of the classical inverse techniques and which are the challenges associated with its application. Both ANN and ELM are considered in the following, highlighting the main differences.

3.1. Application of classical ANN approach to reactor kinetic inversion

The artificial neural networks are biologically inspired computational tools which can flexibly learn from a training sets and offers very powerful generalization properties (Hagan et al., 1996). The application of ANNs for reactor kinetics inversion was proposed in (Picca et al., 2008, 2009, 2010) and consists in the