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Adaptive robust optimal reactive power dispatch in unbalanced distribution networks with high penetration of distributed generation

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Abstract: Due to the increasing penetration of the uncertain single-phase distributed generations (DGs), the current distribution networks are becoming more uncertain, unbalanced, and complicated than ever before, which brings great challenges for distribution network operators. This study proposes an adaptive robust optimal reactive power dispatch approach for the unbalanced distribution networks (U-DNs) considering uncertainty caused by DGs. Leveraging the reactive power compensation from inverters of DGs, the purpose of the proposed method is to minimise power losses and maintain the voltage within regulatory limits. The feasible region of DGs is estimated and considered as a new constraint, aiming to guarantee the reliable operation of U-DNs. The optimal reactive power dispatch problem is then formulated as an adaptive robust optimisation problem based on semidefinite programming. The adaptive function, which derives the relationship between reactive power and active power outputs of DGs, is utilised to make the method more flexible and less conservative. The cutting plane algorithm is introduced to solve the proposed adaptive robust reactive power dispatch model efficiently. Moreover, case studies are separately conducted on the modified IEEE 13-bus and 123-bus test systems to demonstrate the effectiveness of the proposed method.

1 Introduction

Power distribution systems have continued to witness transformations in generation, consumption and operational landscapes. Due to sustainability concerns [1], the penetration of distributed renewable generation, plug-in hybrid electric vehicles, responsive demand [2–4], has increased dramatically. This trend brings the new challenges to the operations of distribution systems including bilateral power flow, uncertainty and single-phase power source integration etc., which may reduce power supply reliabilities, worsen power quality, and increase the three-phase unbalance factor. Meanwhile, the inverters of distributed generations (DGs) are capable of providing the reactive power to facilitate the operation of distribution networks [5, 6]. Hence, this paper proposes a novel approach to reduce the operational power losses and maintain the system voltage within regulatory limits by controlling inverters of DGs.

Numerous efforts have been made for the reactive power dispatch in DNs. The optimal power flow (OPF) is one of the typical and widely used approaches [7, 8]. In OPF, the controlling strategy is given by solving the optimisation mathematical model with objectives, e.g. minimising power losses, minimising voltage deviations from references etc. However, OPF is based on the steady state of DNs, and it is difficult to deal with the unexpected state variation during operations. Some other works focus on controlling the inverter of DGs to support system operation based on sensitivity. This controlling approach has the fast response speed and is applicable for different operating conditions. In [9, 10], the voltage controllers based on sensitivity and Lyapunov function were studied. Calderaro *et al.* [11] proposed the control approach to reduce power losses with maintaining voltages in acceptable limits. As one typical DG, the distributed energy storage system was applied to improve voltage profile in [12, 13]. Ranamuka *et al.* [14] focused on combing DGs with conventional devices to regulate voltage in DNs.

In practice, the phases unbalanced effect as another challenge exists. Due to the non-equilateral conductor spacings of distribution lines, unequal loads in single-phase and the users' single-phase connections, the modern DNs are unbalanced [15]. Moreover, reactive power injections have a counter-intuitive effect on the bus voltage magnitudes across phases. From [16], reducing reactive power injections in one phase may lead the over-voltage phenomenon on other phases. Thus, the single-phase model is not applicable and the ORPD problem is more challenging in unbalanced distribution networks (U-DNs). In [17, 18], DGs and on-load tap changer transformers (OLTCs) are coordinately utilised to control the voltage in U-DNs. Fallahzadeh-Abarghouei *et al.* [19] designed an online voltage management framework based on the network partitioning for U-DNs. Robbins *et al.* [20] and Zhang *et al.* [21] casted the ORPD problem of U-DNs as an optimisation programme to minimise power losses.

However, the accuracy of DG's power output prediction was not considered in the works above. Currently, the prediction accuracy is limited especially in the long-time cycle. The deviation between the predicted and real values of DG's supply is featured as the uncertainty and must be considered in advance. The robust optimisation (RO) [22] considering the uncertainty is effective in solving optimal power dispatch for DNs with high penetration of renewables [23, 24]. Ding *et al.* [23] proposed a robust optimal dispatch method of photovoltaic (PV) inverters under uncertainty. Gao *et al.* [24] designed one robust coordinated active and reactive scheduling system. However, since RO is designed based on the worst case with low probability, its optimal solution is usually considered as conservative [25, 26]. The adjustable uncertainty budget is then utilised to reduce the conservation of conventional RO in the wind power dispatch problem [25] and the economic dispatch problem [26].

In light of the above issues, this paper proposes a novel flexible and low conservative adaptive robust optimal reactive power dispatch (AR-ORPD) method for U-DNs. Three features need to be noted here: (i) This approach holds high robustness, appearing it guarantees all the constraints satisfied in uncertain environments; (ii) the AR-ORPD is high flexible and low conservative as the optimal solution being adaptive to the real-time varying uncertain variable; (iii) the approach is applicable in U-DNs. The first feature is due to a feasible region estimation and a model designation based on the RO theory. In the feasible region estimation, the voltage constraint is directly converted into DG's reactive power supply constraint, which determines the allowable reactive power injection scope under uncertainty. The second feature utilises an adaptive function to replace the definite solution of RO. Since the adaptive function describes the relationship between optimal strategy and real-time value of uncertain variable, the solution of the proposed AR-ORPD depends on the specific case during operation instead of the worst one. The third feature is realised by utilising the rank-relaxed semidefinite programming (SDP) in mathematical modelling.

Major contributions are concluded as follows: (i) a feasible region of DG's reactive power supply is estimated considering uncertainty; (ii) an adaptive RO method is proposed to ease the conservation of traditional RO; (iii) an optimal reactive power dispatch model for U-DNs is constructed to reduce the power loss and maintain the voltage within its regulatory limits.

This paper is organised as follows: Section 2 discusses the general ORPD model. Section 3 introduces the feasible region of U-DNs and formulates the adaptive RO model with the consideration of uncertainty. Section 4 proposes the cutting plane method to solve the optimisation problem. Simulation results are shown in Section 5 to illustrate the performance. Conclusions are drawn in Section 6.

2 Problem formulation

In this paper, the proposed model is formulated in the three-phase system instead of single-phase system. Due to the coupling among phases, the adaptive RO is more complex and challenging compared to the single-phase system.

2.1 Unbalanced distribution networks model

For an unbalanced distribution network (U-DN) with n nodes, all the nodes are indexed as {1,2, …, n} by the number set $N = \{1, 2, \ldots, n\}$ while the phase of each node is expressed by φ, $\varphi \in \{a, b, c\}$. Similarly, the nodes connected with DGs are denoted as N_{DG}. For an arbitrary line from node i to node k, it is represented as (i,k). For an arbitrary node i at phase φ, the bus voltage is denoted as V_i^φ and the power injection is denoted as $S_i^\varphi = P_i^\varphi + \mathrm{j}Q_i^\varphi$, where P_i^φ and Q_i^φ describe the active and reactive power injection, respectively. For a DG integrating through phase φ of node i, its complex power output is represented as $S_{i,\mathrm{DG}}^\varphi = P_{i,\mathrm{DG}}^\varphi + \mathrm{j}Q_{i,\mathrm{DG}}^\varphi$, where $i \in N_{\mathrm{DG}}$. The complex power of load at phase φ of bus i is defined as $S_{i,\mathrm{load}}^\varphi = P_{i,\mathrm{load}}^\varphi + \mathrm{j}Q_{i,\mathrm{load}}^\varphi$.

When the system is coupled in the three-phase model, the admittance is given as $\boldsymbol{y}_{ik}^{\Phi\Phi} = \boldsymbol{g}_{ik}^{\Phi\Phi} - \mathrm{j}\boldsymbol{b}_{ik}^{\Phi\Phi}$ while the impedance is shown as $z_{ik}^{\Phi\Phi} = \boldsymbol{r}_{ik}^{\Phi\Phi} + \mathrm{j}\boldsymbol{x}_{ik}^{\Phi\Phi}$. Let $\boldsymbol{Y}^{\Phi\Phi} \in \mathbb{C}^{3n\times 3n}$ denote the admittance matrix. To bus i, the three-phase voltage vector is shown as $\boldsymbol{V}_i^\Phi := [V_i^a, V_i^b, V_i^c]^{\mathrm{T}}$ while complex power injection vector, active power injection vector and reactive power injection vector are shown as, $\boldsymbol{S}_i^\Phi := [S_i^a, S_i^b, S_i^c]^{\mathrm{T}}$, $\boldsymbol{P}_i^\Phi := [P_i^a, P_i^b, P_i^c]^{\mathrm{T}}$ and $\boldsymbol{Q}_i^\Phi := [Q_i^a, Q_i^b, Q_i^c]^{\mathrm{T}}$, respectively. Correspondingly, the bus voltage vector is described as $\boldsymbol{V}^\Phi = [\boldsymbol{V}_1^\Phi, \boldsymbol{V}_2^\Phi, \ldots, \boldsymbol{V}_n^\Phi]$.

Based on the U-DN model, the power injected at bus i is shown as

$$\boldsymbol{S}_i^\Phi = \mathrm{diag}(\boldsymbol{V}_i^\Phi) \sum_{k \in E_i} [\boldsymbol{Y}^{\Phi\Phi}]_{ik}^*(\boldsymbol{V}_k^\Phi)^* \tag{1}$$

where

$$\mathrm{diag}(\boldsymbol{V}_i^\Phi) = \begin{bmatrix} V_i^a & 0 & 0 \\ 0 & V_i^b & 0 \\ 0 & 0 & V_i^c \end{bmatrix}.$$

2.2 Optimal reactive power dispatch in unbalanced distribution networks

The SDP optimisation is a well-established method in OPF problem for unbalanced systems [20, 21] and is applied in this paper to establish the optimal reactive power dispatch mathematical model. Firstly, we define $\boldsymbol{W} = \boldsymbol{V}^\Phi(\boldsymbol{V}^\Phi)^{\mathrm{H}}$, and $\boldsymbol{W}$ is one positive semidefinite matrix with rank 1. The $\boldsymbol{e}_i^\varphi$ is defined as the vector which only has one non-zero entry set to 1 corresponding to phase φ of bus i.

The $\boldsymbol{A}_i^\varphi = (\boldsymbol{Y}^{\Phi\Phi})^{\mathrm{H}}\boldsymbol{E}_i^\varphi$ is defined with $\boldsymbol{E}_i^\varphi = \boldsymbol{e}_i^\varphi(\boldsymbol{e}_i^\varphi)^{\mathrm{T}}$. Thus, the complex power injection at phase φ of bus i can be rewritten as

$$\boldsymbol{S}_i^\varphi = \mathrm{Tr}(\boldsymbol{A}_i^\varphi \boldsymbol{W}) \tag{2}$$

The objective function to reduce the power losses is written as

$$f(\boldsymbol{W}) = \sum_{i \in N} \sum_{\varphi = a}^{c} \mathrm{Re}\{Tr(\boldsymbol{A}_i^\varphi \boldsymbol{W})\} \tag{3}$$

The constraints are set as follows:

(a) *Power flow balance constraint:*

$$\mathrm{Tr}(\boldsymbol{A}_i^\varphi \boldsymbol{W}) - \boldsymbol{S}_i^\varphi = 0 \quad \forall i \in N \setminus N_{\mathrm{DG}} \tag{4}$$

$$\mathrm{Tr}(\boldsymbol{A}_i^\varphi \boldsymbol{W}) - \boldsymbol{S}_{i,\mathrm{DG}}^\varphi + \boldsymbol{S}_{i,load}^\varphi = 0 \quad \forall i \in N_{\mathrm{DG}} \tag{5}$$

(b) *Voltage limitations:*

$$V^{\min} \le |V_i^\varphi| \le V^{\max} \tag{6}$$

where $V^{\min}$ and $V^{\max}$ represent the lower and upper limit of node voltage, respectively.

(c) *Capacity constraints of DG:*

For an arbitrary DG, the capacity constraint is shown as

$$(p_{i,DG}^\varphi)^2 + (q_{i,DG}^\varphi)^2 \le (s_{i,DG}^\varphi)^2 \tag{7}$$

where $p_{i,DG}^\varphi$ is the active power output, $q_{i,DG}^\varphi$ is the reactive power output and $s_{i,DG}^\varphi$ is the capacity of DG, respectively.

(d) Reactive *power constraints of DG:*

$$-Q_{i,DG}^{\varphi,\max} \le q_{i,DG}^\varphi \le Q_{i,DG}^{\varphi,\max} \tag{8}$$

where $Q_{i,DG}^{\varphi,\max}$ represents the maximum value of DG's reactive power supply.

3 Adaptive robust optimal reactive power dispatch model

3.1 Uncertainty of DGs

The DG's power supply is uncertain and expressed as an interval set. The active power injection of DG connected to phase φ of node i is shown as

$$p_{i,DG}^\varphi = [\underline{p_{i,DG}^\varphi}, \overline{p_{i,DG}^\varphi}] \tag{9}$$

where $\underline{p_{i,DG}^\varphi}$ and $\overline{p_{i,DG}^\varphi}$ are the lower and upper bound of active power injection of DG at phase φ of node i.

By the equal transformation, the interval set (9) can be transformed into an affine form, shown as