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Possible acception criteria for structure functions

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ABSTRACT

This paper deals with the verification of thermal transient evaluation implementations. This subject is relevant because e.g. the upcoming standard will describe the thermal transient measurement as a standard method to estimate the junction-to-case thermal resistance [1,2], thus anybody can create their own implementation of the evaluation method. We have to have a method to verify these implementations. For this reason we examined the result of the NID (Network Identification by Deconvolution) method from different aspects. For these examinations we defined a multilayer structure as a reference structure and we analytically expressed the unit-step response and the cumulative structure function of this structure. Using the unit-step response as an input data set for the implementation in question we got an approximation of the structure function. Analysing this and the reference RC network we could define a practical maximum tolerance for the deviation between the analytical and the calculated functions.

1. Introduction

In this paragraph we summarize the definition of network representing functions, based on [3]. Owing to size limitations we do not present here the detailed discussions and proofs concerning these notions and their relations. Nevertheless, if the reader accepts the equations presented in this chapter, following the further parts of the paper should not raise difficulties. Should a deeper insight into the theoretical background be demanded [3,4] can be examined.

A lumped element one-port can be represented by a finite number of τ time-constants and R magnitudes. The port-impedance of a lumped element network has discrete "spectrum lines" in finite number. An infinite distributed network (e.g. a thermal system) has no discrete lines, it can be described with the help of a continuous time-constant spectrum. The physical meaning of this idea is that in a general response any time-constant can occur in some amount, some density, so a density spectrum may suitably represent it. For practical reasons we use logarithmic variable for the time and the time-constants:

$$z = \log t, \quad \zeta = \log \tau \tag{1}$$

Let us consider a distributed RC one-port, the response of which contains numerous exponentials having different time-constants and magnitudes. The "time-constant density" is defined as

$$R(\zeta) = \lim_{\Delta\zeta \to 0} \frac{\text{sum of magnitudes between } \zeta \text{ and } \zeta + \Delta\zeta}{\Delta\zeta} \tag{2}$$

From this definition directly follows the fact that the step-function response can be composed from the time-constant density:

$$a(t) = \int_{-\infty}^{\infty} R(\zeta) \cdot [1 - e^{-t/e^{\zeta}}]\, d\zeta \tag{3}$$

This integral is the generalisation of the step-function response of a lumped element network. Using the logarithmic time variable in the integral of Eq. (3)

$$a(z) = \int_{-\infty}^{\infty} R(\zeta) \cdot [1 - e^{-e^{z-\zeta}}]\, d\zeta \tag{4}$$

a convolution-type integral equation is obtained. Differentiating both sides with respect to z, we obtain

$$\frac{d}{dz} a(z) = R(z) \otimes W(z) \tag{5}$$

where

$$W(z) = e^{z - e^{z}} \tag{6}$$

The $W(z)$ weight function can be seen in the Fig. 1.

For the practical calculation of the time-constant density function, we use the relation between the $Z(s)$ complex impedance function and $R(\zeta)$:

$$R(\zeta) = \frac{1}{\pi} \operatorname{Im}(Z(s = -e^{-\zeta})) \tag{7}$$

The proof of Eq. (7) can be found in Ref. [3]. Eq. (7) suggests that only the $j\omega$ imaginary frequency has to be replaced by the

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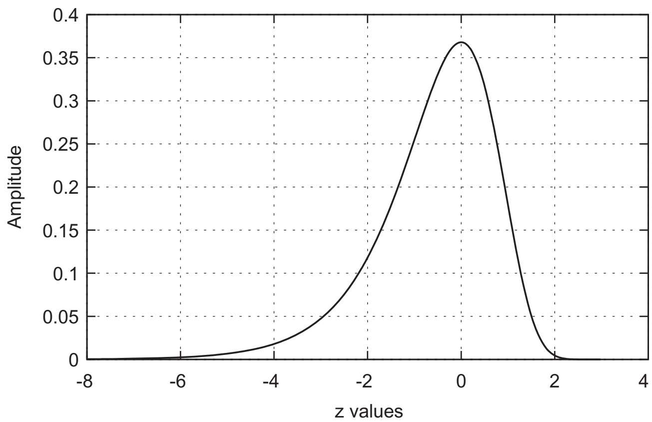

Fig. 1. The $W(z)$ weight function in the convolution equation (5).

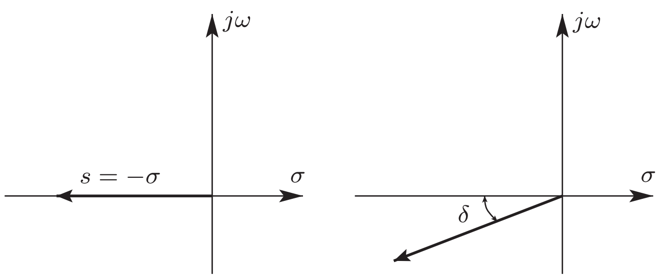

Fig. 2. The $s(z)$ line on the complex plane from Eq. (8).

$s=-\exp(-z)$ complex frequency and then the imaginary part of the calculated complex response multiplied by $1/\pi$ provides the time-constant spectrum.

However, the procedure is not as straightforward as it seems because of the use of Eq. (7). This equation requires a great amount of caution. As the equation shows the imaginary part of the Z impedance has to be calculated along the negative real axis of the complex plane. Along this axis, singularities usually lie: the poles of the network equation of lumped circuits or some singular lines in the case of distributed systems. These singularities can prevent the use of Eq. (7) for the calculation of the time-constant spectrum.

We can overcome these difficulties by adopting an approximate solution. In order to bypass the "dangerous" area, we have to avoid following the negative real axis (Fig. 2). A line that is appropriately close to this axis might be used instead [9], such as

$$s=-(\cos\delta+j\sin\delta)e^{-z} \tag{8}$$

Obviously, the δ angle has to be very small, not more than 2°–5°. Even if this angle is small, an error is introduced into the calculation. It can be proven that the calculated $R_C(z)$ time-constant spectrum can be expressed with the exact one by the following convolution equation:

$$R_C(z)=\frac{\pi-\delta}{\pi}R(z)\otimes e_r(z) \tag{9}$$

where

$$e_r(z)=\frac{1}{\pi-\delta}\frac{\sin\delta e^{-z}}{1-2\cdot\cos\delta e^{-z}+e^{-2z}} \tag{10}$$

This function is a narrow pulse of unity area. The error of the calculation is represented by this function. Diminishing δ the $e_r(z)$ function becomes narrower and narrower. Thus, any accuracy requirement can be fulfilled by choosing an appropriately small δ angle. The half-value width, which is a measure of the resolution, is given by

$$\Delta_e=2\ln\left(2-\cos\delta+\sqrt{(2-\cos\delta)^2-1}\right)\simeq 2\delta \tag{11}$$

If, for example, $\delta=2°$, then the resolution is 0.1 octave, which means that two poles can be distinguished if the ratio between their frequencies is greater than 1.072.

2. The process of the comparison

The process of the comparison can be seen in Fig. 3.

First, we have to define a reference structure, where we know every necessary parameters. For practical reason we defined a multilayer structure, where there are more than one significant time-constant components. The model of the reference structure can be seen in Fig. 4. It is well known [5] that the $Z(s)$ impedance of a uniform transmission line can be calculated based on the telegraph equations:

$$Z_{in}=Z_0\frac{Z_t\cdot\cosh\gamma L+Z_0\cdot\sinh\gamma L}{Z_t\cdot\sinh\gamma L+Z_0\cdot\cosh\gamma L} \tag{12}$$

where Z_t is the termination of the back side, L is the length of the transmission line, Z_{in} is the input impedance and

$$\gamma=\sqrt{s\cdot rc},\quad Z_0=\sqrt{\frac{r}{s\cdot c}} \tag{13}$$

where r and c are the unit length thermal resistance and capacitance of the material. If we have more than one transmission lines, we can use Eq. (12) on each section, where the input impedance of one line is the termination impedance of the previous line. We use Eq. (12) recursively (Table 1).

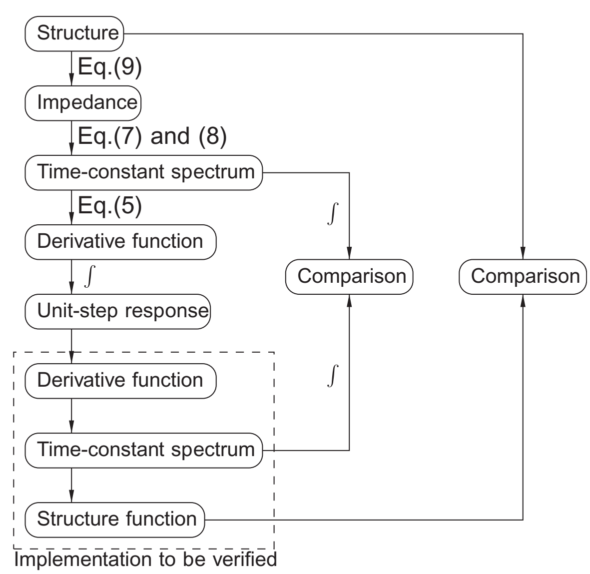

Fig. 3. The process of the comparison, the numbers means which equations are used in the step.

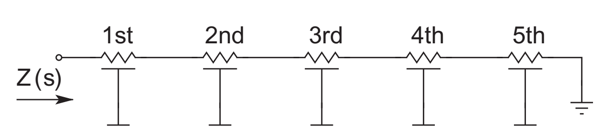

Fig. 4. The multilayer distributed RC model of the reference structure.