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Continuous fractional-order grey model and electricity prediction research based on the observation error feedback

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ABSTRACT

As superiority to conventional statistical models, the grey model based on the fractional calculus becomes a hot topic and show great potentials with excellent performance. In this paper, the generalized fractional-order forms for grey models are given, which could have more freedom and better modeling by the fractional derivatives. The case of per capita output of electricity prediction is discussed by the modified optimized fractional grey model using the error feedback. The performance is evaluated and greatly improved in modeling and prediction compared with some traditional grey methods. Due to the fractional derivatives, the novel model could provide the fitting, prediction with more freedom and enrich the content, scope and application of grey theory.

1. Introduction

As industrial production occupies a large proportion in the national economy of China, many industrial products are used to get the static sampling for data mining. Among these, energy is the foundation of economic development, per capita output of electricity is an important indicator and has great relationship with people's life and country's economic development. Therefore, the electricity consumption forecasting has become a challenging task for electric utilities.

Various tools and techniques have been studied for electricity consumption and load forecasting problems, which provide more choices when obtaining models. e.g., artificial neural network (ANN), support vector regression (SVR), expert systems, machine learning, soft computing, random forest, iterative reweighted least-squares, fuzzy logic, fuzzy inductive reasoning, continuous wavelet models, logistic models, multiple linear regression, autoregressive integrated moving average (ARIMA) models, generalized autoregressive conditional heteroskedasticity (GARCH), exponential smoothing models, grey system and some hybrid models [1–9].

Grey system theory (GST), founded by Deng Julong, includes grey system analysis, modeling, prediction, decision-making, control and so on. It becomes a useful tool in processing uncertain system with small samples and poor information [10,11]. Grey prediction is an important embranchment in GST and has been widely used in forecasting by grey models (GMs). Compared with other forecasting methods, only a limited amount of data are needed to estimate the behavior of unknown systems without knowing the mathematical model in the grey system. Therefore, The main purpose of the theory is to predict the behavior of systems which cannot be detected by stochastic or fuzzy methods with limited data [12].

During the past years, many new forms were proposed to achieve effective forecasting in the GM research, including extended, optimized generalized and modified GMs. The fractional calculus describes a real model or process more accurately than the classical “integer-order” ones, and GM based on the fractional calculus becomes a hot topic. At present, the fractional GM researches mainly focus on the following aspects, the fractional accumulation of discrete GM and its disturbance problem [13–26], continuous fractional-order GM [27,28]. Wu Lifeng, Liu Sifeng, Xiao Xinping, Mao Shuhua and other researchers proposed several models based on the fractional accumulation, for example, FAGM(1,1) (fractional order accumulation grey model), GGM(1,1) (generalized accumulation grey model), $NDGM^{\frac{p}{q}}$ (1,1) (non-homogenous discrete grey model), FADGEM (fractional-order accumulation discrete grey

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exponential model) and GM(1,N,τ) (fractional-order accumulation time-lag grey model) and reverse accumulation model. Among these, the fractional operator is mainly manifested in the pre-treatment of data rather than in the model structure, the bleaching equation remains the same as the integer-order one. For the continuous fractional-order GM, Mao Shuhua et al. established a grey prediction model by Caputo fractional derivative, defined as $\frac{d^q x^{(r)}}{dt^q} + ax^{(r)} = b$[27], Wu Lifeng et al. proposed a FGM (q, 1), where the differential equation is expanded for the fractional-order one, and differential numerical solution of the model is obtained [28]. The comparison between some GM forecasting methods by fractional calculus can be seen in Table 1 and general continuous fractional-order GMs modeling and prediction researches have not been conducted.

The fractional GMs are researched generally and the aim of this study is to present a new methodology for electricity forecasting using a modified GM. Grey prediction only needs small amount of data for reliable and acceptable accuracy, which is one of the main advantages of grey prediction over other methods [29]. Meanwhile, the fractional GMs may be considered as generalized models for classical ones, and more freedom and flexibility are added for the new models using small amount of data. In order to improve the robustness of the system for modeling and prediction, parameters can be optimized and the modified model can further be used, where the history output and the error of system based on the feedback are considered.

The paper is organized as follows: in Section 2, the basic knowledge of fractional calculus which will be used in the GM modeling is introduced. In Section 3, the forms of fractional-order GMs and the modified model are given. In Section 4, the fractional-order grey modeling analysis is given and analyzed. In Section 5, the example of modified optimized fractional-order GM is discussed and compared with some typical models. The conclusion part is given finally.

2. Fractional calculus

Fractional calculus plays an important role in modern science and is a generalization of differentiation and integration to non-integer-order fundamental operator. The Grünwald-Letnikov (G-L) definition is given as

$$ {}_aD_t^{\gamma} f(t) = \frac{d^{\gamma} f(t)}{dt^{\gamma}} = \lim_{h\to 0} \frac{1}{h^{\gamma}} \sum_{j=0}^{\frac{t-a}{h}} (-1)^j \binom{\gamma}{j} f(t-jh) \tag{1} $$

where ${}_aD_t^{\gamma}$ is the continuous differential-integral operator, a is the initial value, h is the sampling time,γ is non-integer and can be positive or negative value corresponding to differentiation and integration respectively, $\binom{\gamma}{j} = \frac{\gamma!}{j!(\gamma-j)!}$. For the single input single output (SISO) problem with zero initial conditions, the fractional-order system can be described as

$$ \begin{aligned} & a_{n0} D_t^{\alpha_n} y(t) + a_{n-10} D_t^{\alpha_{n-1}} y(t) + \ldots + a_{00} D_t^{\alpha_0} y(t) \\ & = b_{m0} D_t^{\beta_m} u(t) + b_{m-10} D_t^{\beta_{m-1}} u(t) + \ldots + b_{00} D_t^{\beta_0} u(t) \end{aligned} \tag{2} $$

where $a_i(i = 0,1,\ldots,n)$ and $b_j(j = 0,1,\ldots,m)$ are the coefficient numbers, $\alpha_i(i = 0,1,\ldots,n)$ and $\beta_j(j = 0,1,\ldots,m)$ are the fractional-order numbers with real or rational ones. Without losing generality, it is here assumed that $\alpha_n > \alpha_{n-1} > \ldots > \alpha_0$ and $\beta_m > \beta_{m-1} > \ldots > \beta_0$. The Laplace transform of the fractional derivative with left-inverse operator is:

$$ L(D^{\gamma} f(t)) = s^{\gamma} F(s) - \sum_{k=0}^{m-1} D^k J^{(m-\gamma)} f\left(0^+\right) s^{m-1-k}, \quad m-1 \le \gamma \le m \tag{3} $$

The fractional-order transfer function for equation (2) can be expressed as

$$ G(s) = \frac{Y(s)}{U(s)} = \frac{b_m s^{\beta_m} + b_{m-1} s^{\beta_{m-1}} + \ldots + b_0 s^{\beta_0}}{a_n s^{\alpha_n} + a_{n-1} s^{\alpha_{n-1}} + \ldots + a_0 s^{\alpha_0}} \tag{4} $$

The numerical solution of the system with zero initial condition in equation (2) is [30].

$$ y(t) = \frac{\hat{u}(t) - \sum_{i=0}^{n} \frac{a_i}{h^{\alpha_i}} \sum_{j=1}^{\frac{t-a}{h}} (-1)^j q_j^{(\alpha_i)} y(t-jh)}{\sum_{i=0}^{n} \frac{a_i}{h^{\alpha_i}}} \tag{5} $$

where $\hat{u}(t) = b_m D_t^{\beta_m} u(t) + b_{m-1} D_t^{\beta_{m-1}} u(t) + \ldots + b_0 D_t^{\beta_0} u(t)$ and it should be evaluated first using the same method mentioned in equation (4). $q_j^{(\alpha_i)}$ is the binomial coefficients and can be calculated from $q_0^{(\alpha_i)} = 1,\ q_j^{(\alpha_i)} = \left(1 - \frac{1+\alpha_i}{j}\right) q_{j-1}^{(\alpha_i)}$.

The two-parameter Mittag-Leffler function is defined as [31]:

$$ E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(k\alpha+\beta)} \tag{6} $$

where $\mathrm{Re}(\alpha) > 0$, $\beta \in \mathbb{C}$, $z \in \mathbb{C}$ and equation (6) can be also expressed in the integral form as equation (7) shows

$$ E_{\alpha,\beta}(z) = \frac{1}{2\pi i} \int_C \frac{t^{\alpha-\beta} e^t}{t^{\alpha} - z} dt \tag{7} $$

The contour C starts and ends at $-\infty$ and circles around the singularities and branch points of the integrand.

Table 1
Comparison between some GM forecasting methods by fractional calculus.

Model	Definition	Parameter description
FAGM(1,1)	$x^{\left(\frac{p}{q}\right)}(k) - x^{\left(\frac{p}{q}\right)}(k-1) + az^{\left(\frac{p}{q}\right)}(k) = b$	$x^{\left(\frac{p}{q}\right)}$ is called as the $\frac{p}{q}$ th order accumulated generating operator and $x^{\left(\frac{p}{q}\right)}(k) = \sum_{i=1}^{k} C_{k-i+\frac{p}{q}-1}^{k-i}$
GGM(1,1)	$x^{(r)}(k) - x^{(r)}(k-1) + az^{(r)}(k) = b$	$x^{(r)} = A^r x^{(0)}, z^{(r)}(k) = \alpha x^{(r)} + (1-\alpha) x^{(r)}(k-1), \alpha \in (0,1)$
NDGM$^{\frac{p}{q}}$ (1,1)	$x^{\left(\frac{p}{q}\right)}(k+1) = \beta_1 x^{\left(\frac{p}{q}\right)}(k) + \beta_2(k) + \beta_3$	$x^{\left(\frac{p}{q}\right)}$ is the $\frac{p}{q}$ th order accumulating generated sequence
FADGEM	$x^{(r)}(k+1) = \beta_0 + \beta_1 k^{\gamma} + \beta_2 x^{(r)}(k)$	r represents accumulating generated operator
GM(1,N,τ)	$\frac{dy^{(r)}}{dt} + ay^{(r)} = \sum_{i=1}^{N-1} b_i x_i^{(r)}(t-\tau)$	r represents accumulating generated operator
FGM(q,1)	$\frac{d^q x^{(r)}}{dt^q} + ax^{(r)} = b$	q is the fractional derivative, $x^{(r)}$ is the r th order accumulating generated sequence
NIGM(p,1)	$a_{(1)} x_{(1-p)}(k) + az^{(0)}(k) = b$	$a^{(1)} x^{(1-p)}(k) = x^{(1-p)}(k) - x^{(1-p)}(k-1)$