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Energy conversion under conjugate conduction, magneto-convection, diffusion and nonlinear radiation over a non-linearly stretching sheet with slip and multiple convective boundary conditions

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ABSTRACT

Energy conversion under conduction, convection, diffusion and radiation has been studied for MHD free convection heat transfer of a steady laminar boundary-layer flow past a moving permeable non-linearly extrusion stretching sheet. The nonlinear Rosseland thermal radiation flux model, velocity slip, thermal and mass convective boundary conditions are considered to obtain a model with fundamental applications to real world energy systems. The Navier slip, thermal and mass convective boundary conditions are taken into account. Similarity differential equations with corresponding boundary conditions for the flow problem, are derived, using a scaling group of transformation. The transformed model is shown to be controlled by magnetic field, conduction-convection, convection-diffusion, suction/injection, radiation-conduction, temperature ratio, Prandtl number, Lewis number, buoyancy ratio and velocity slip parameters. The transformed non-dimensional boundary value problem comprises a system of nonlinear ordinary differential equations and physically realistic boundary conditions, and is solved numerically using the efficient Runge-Kutta-Fehlberg fourth fifth order numerical method, available in Maple 17 symbolic software. Validation of results is achieved with previous simulations available in the published literature. The obtained results are displayed both in graphical and tabular form to exhibit the effect of the controlling parameters on the dimensionless velocity, temperature and concentration distributions. The current study has applications in high temperature materials processing utilizing magnetohydrodynamics, improved performance of MHD energy generator wall flows and also magnetic-microscale fluid devices.

1. Introduction

The investigation of transport problems of viscous fluids associated with energy conversion passing a moving/stationary linearly/non-linearly extrusion surface is a relevant problem in many industrial processes including manufacture, drawing of plastics and rubber sheets, glass fiber and paper production, metal and polymer extrusion processes, cooling of metallic sheets and crystal growth, all of which utilize excessive energy input. It is necessary to cool the extrusion stretching sheet when the manufacturing process at high temperature. These flows need viscous fluids to make a good effect to control excessive temperature in the sheet. In addition, the fluids have been processed using a variety of supplementary effects (i.e. magnetic force, thermal/mass buoyancy and mass diffusion) for the problem, and effectively such systems constitute a conjugate energy conversion system which for optimization, requires both experimental and theoretical analysis. The rate of cooling/heating can be instrumental in determining the constitution of manufactured materials, in which a moving surface emerges from a slit and consequently, a boundary layer flow adjacent to the sheet is generated in the direction of the movement of the surface. Sakiadis [1] first investigated the boundary flow past a continuous solid surface, motivated by chemical processing applications. Thereafter Crane [2] studied the steady two-

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Nomenclature

a	velocity slip parameter (−)
$B(\bar{x})$	local magnetic field strength (T)
B_0	magnitude of magnetic field strength (T)
C	concentration (kgmol/m^3)
C_w	wall concentration (kgmol/m^3)
C_∞	ambient concentration (kgmol/m^3)
c_p	specific heat at constant pressure (J/kg K)
D	diffusion coefficient (m^2/s)
$f(\eta)$	dimensionless stream function (−)
fw	suction/injection parameter (−)
g	acceleration due to gravity (m/s^2)
h_f	heat transfer coefficient (W/m^{2}K)
h_m	mass transfer coefficient (m/s)
k	thermal conductivity (m^2/s)
k_1	Rosseland mean absorption coefficient (1/m)
L	characteristic length (m)
m	power law index of wall temperature and concentration (−)
M	magnetic field parameter (−)
N	radiation-conduction parameter (−)
$N_1(\bar{x})$	local velocity slip factor (s/m)
$(N_1)_0$	constant velocity slip factor (s/m)
Nc	convection-conduction parameter (−)
Nd	convection-diffusion parameter (−)
$Nu_{\bar{x}}$	local Nusselt number (−)
Pr	Prandtl number (−)
p	pressure (N/m^2)
q_m	wall mass flux (kg/s m^2)
q_w	wall heat flux (W/m^2)
q_r	component of radiative heat flux in $\bar{y}$- direction (W/m^2)
Le	Lewis number (−)
Ra	Rayleigh number (−)
$Sh_{\bar{x}}$	local Sherwood number (−)
T	fluid temperature (K)
T_w	wall temperature (K)
T_r	temperature ratio parameter (−)
T_∞	ambient temperature (K)
$\bar{u}, \bar{v}$	velocity components along the $\bar{x}$− and $\bar{y}$− axes (m/s)
$\bar{u}_w$	sheet velocity (m/s)
$\bar{v}_w$	transpiration velocity (m/s)
$\bar{x}, \bar{y}$	Cartesian coordinates along and normal to the sheet (m)
Greek	
α	thermal diffusivity (m^2/s)
β_T	volumetric thermal expansion coefficient (1/K)
β_C	volumetric mass expansion coefficient (m^3/kgmol)
η	similarity variable (−)
$\theta(\eta)$	dimensionless temperature (−)
μ	viscosity of the fluid (Ns/m^2)
ν	kinematic viscosity of the fluid (m^2/s)
ρ	fluid density (kg/m^3)
σ_0	constant electric conductivity (S/m)
σ_1	Stefan-Boltzmann constant (W/m^2-K$^{4)}$
$\varphi(\eta)$	dimensionless concentration (−)
ψ	stream function (−)

dimensional boundary layer flow of a viscous, incompressible fluid induced by a stretching sheet. As pointed out by Wang [3], there have been numerous analytical and numerical studies communicated on stretching/shrinking sheet flows. In this context we quote Pantokratoras [4], Van Gorder et al. [5], Hayat et al. [6] and Noghrehabadi et al. [7]. These studies have explored a wide range of thermophysical effects in stretching sheet transport phenomena. Yao et al. [8] reported on heat transfer of a viscous fluid flow past a stretching/shrinking sheet with a convective boundary condition. Bachok et al. [9] examined stagnation point flow toward a stretching/shrinking sheet with a convective surface boundary condition. Some recent studies related to energy conversion are the paper of Elshafei [10] who studied natural convection heat transfer from a heat sink with hollow/perforated circular pin fins. Sertkaya et al. [11] presented pin-finned surfaces in natural convection. Bouaziz and Aziz [12] studied convective–radiative fin with temperature dependent thermal conductivity using double optimal linearization. Jang et al. [13] studied 3-D turbulent flow of venting flue gas using thermoelectric generator modules and plate fin heat sink. Torabi et al. [14] studied longitudinal fins of rectangular, trapezoidal and concave parabolic profiles with multiple nonlinearities.

Magnetohydrodynamics (MHD) has also grown into a significant area in many branches of engineering, not least in sustainable alternative energy generation. MHD involves the study of the influence of a magnetic field on the viscous flow of electrically-conducting fluids. It arises in magnetic materials processing, purification of crude oil, magnetohydrodynamic electrical power generation, manipulation of electro-conductive polymers, smart braking systems, external aerodynamic flow control for spacecraft and is also critical to TOKAMAK energy systems. In modern electromagnetic materials processing, MHD transport phenomena are exploited frequently in flows from continuously moving, stretching/shrinking, heated/cooled surfaces in a quiescent/moving free stream (Bataller [15]). MHD achieves excellent modification and control of magnetic fluids, which can be synthesized for specific applications including aerospace alloys (Beg et al. [16]). The manufactured materials are affected by the rate of stretching/shrinking, wall heat/mass transfer rates as well as by magnetic field strength (Chen [17]). Other uses of MHD include spacecraft landing gear systems (Holt [18]), deep space nuclear powered engines (Rashidi et al. [19]), magnetoplasma dynamic thrusters (Makinde and Bég [20]) and magnetic materials processing (Beg et al. [21]).

Thermal radiation heat transfer is important when the difference between the surface temperature and the free stream temperature is large and when the operating temperature is significantly high. Radiation plays an important role in controlling heat/mass as well as momentum transfer. It therefore exerts a substantial influence on the final constitution of materials during manufacturing, which can markedly impact on time to delivery by manufacturers. High temperature plasmas, cooling of nuclear reactors and glass production are some important applications of radiative heat transfer from a surface to conductive fluids. The effect of radiation on convective heat/mass transfer flow of both Newtonian and non-Newtonian fluids from either linearly or nonlinearly stretching/shrinking sheets has received extensive attention. Important studies in this regard include Chen [22], Noor et al. [23], Cortel [24], Misra and Sinha [25] and Hakeem et al. [26]. Previous investigators applied a linear Rosseland diffusion approximation for radiation which has limited accuracy when the