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Analysis of Alpha Modes in Multigroup Diffusion

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Abstract - *The alpha eigenvalue problem in multigroup neutron diffusion is studied with particular attention to the theoretical analysis of the model. Contrary to previous literature results, the existence of eigenvalue and eigenflux clustering is here investigated without the simplification of a unique fissile isotope or a single emission spectrum. A discussion about the negative decay constants of the neutron precursors concentrations as potential eigenvalues is provided. An in-hour equation is derived by a perturbation approach recurring to the steady state adjoint and direct eigenvalue problems of the effective multiplication factor and is used to suggest proper detection criteria of flux clustering. In spite of to prior work, the in-hour equation results for a necessary and sufficient condition for the existence of the eigenvalue-eigenvector pair. A simplified asymptotic analysis is used to predict bands of accumulation of eigenvalues close to the negative decay constants of the precursors concentrations. The resolution of the problem in one-dimensional heterogeneous problems shows numerical evidence of the predicted clustering occurrences and also confirms previous theoretical analysis and numerical results.*

keywords: *neutron kinetics, alpha modes, clustering.*

I. INTRODUCTION

The kinetic neutron equation with precursors is an efficient tool for the analysis of the neutron time evolution and, therefore, finds multiple applications in reactor control and the study of accident scenarios. The time-dependent flux depends on the initial conditions for neutrons and precursors, the boundary condition for neutrons, the presence of external sources and the time-dependent changes of the cross sections, such as those produced by rod motion and changes of Boron concentration or to small stochastic perturbation induced by the coolant flow and, in the long time, by radioactive decay and nuclide depletion and creation from fission.

However, if the cross sections and boundary conditions remain constant in time and the sources vanish, then the state of the system tends asymptotically in time to an exponential behavior, which is independent of the earlier changes of the system and, specially, of the initial conditions.[1]

The exponential behaviors that a given system can adopt are the solutions of an eigenvalue equation and can be used to describe the fast evolution of the system as well as to characterize the reactivity of the system. The solutions of this equation are known as time-dependent modes or, more simply, alpha modes, where "alpha" refers to the most frequently adopted symbol for these eigenvalues.

Alpha modes have been applied to formally derive different forms of the well-known in-hour equation, to obtain solutions of the kinetic equations by expansion techniques,[2] to develop numerical solution methods[3] and also as weighting fluxes to homogenize the kinetic equation.[4] These applications as well as the theoretical interest of alpha modes have been the object of intense study. However, with the exception of a few scattered mathematical results,[5, 6, 7] a detailed description of these modes have not yet been given for the diffusion equation. The purpose of this paper is to give a detailed analysis of the alpha modes for a slab geometry using multigroup diffusion theory. We give ample numerical evidence for all the modes predicted from mathematical analysis as well as from physical arguments.

General equations for the alpha eigenvalue problem are discussed in Sec. II, including a perturbation expression in terms of reactivity. In Sec. III we summarize our results and observations for the multigroup one-dimensional slab. Some numerical results are illustrated in Sec. IV, while in Sec. V we compare with Asahi's analytical results for the one-group problem.[6] Conclusions follow in Sec. VI.

For simplicity we shall use a notation based on a continuous formulation and let to the reader the change to a fully discretized operator which is used in the numerical application.

II. THE ALPHA EIGENVALUE EQUATIONS

Our starting point are the time-dependent kinetic diffusion equations coupled to the precursors equations in a heterogeneous domain $\mathcal{D}$:

$$\begin{cases} \left(\frac{1}{v}\partial_t + \mathcal{B}_{pr}\right)\psi = \sum_p \chi_p \lambda_p C_p, & \text{(1a)} \\ (\partial_t + \lambda_p)C_p = \mathcal{F}_p \psi. & \text{(1b)} \end{cases}$$

Here $\psi(x,t)$ is the scalar flux, $x = (\mathbf{r}, E)$ stands for the phase space variables, $C_p(\mathbf{r},t)$ is the concentration for precursor p and

$$\mathcal{B}_{pr} = \mathcal{L} - \mathcal{P}_{pr}$$

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