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Original Article

Scattering cross section for various potential systems

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ABSTRACT

We discuss the problems of scattering in this framework, and show that the applied method is very useful in the investigation of the effect of the resonance in the observed scattering cross sections. In this study, not only the scattering cross sections but also the decomposition of the scattering cross sections was computed for the $\alpha-\alpha$ system. To obtain the decomposition of scattering cross sections into resonance and residual continuum terms, the complex scaled orthogonality condition model and the extended completeness relation are used. Applying the present method to the $\alpha-\alpha$ and $\alpha-n$ systems, we obtained good reproduction of the observed phase shifts and cross sections. The decomposition into resonance and continuum terms makes clear that resonance contributions are dominant but continuum terms and their interference are not negligible. To understand the behavior of observed phase shifts and the shape of the cross sections, both resonance and continuum terms are calculated.

1. Introduction

Studies of scattering problems in nuclear physics have been developed using various experimental techniques and theoretical methods. One very promising method, the complex scaling method (CSM) [1], has been applied to scattering and resonance problems. This approach seems to promise to unify the description of the nuclear structure and reactions, also including nuclear data evaluation, especially for light nuclear mass systems [2–6]. In this work, we use the CSM to study scattering phase shifts. It has been shown to be possible to calculate scattering phase shifts according to the continuum level density (CLD) [7]. We develop a method of calculating the CLD to investigate the effects of the resonant states, which are related to the nuclear structures, and which are separate from the continuum states. The background contributions to the phase shifts are also considered. This method is applied to the complex scaled orthogonality condition model [4] of different scattering systems including the $\alpha-n$ and $\alpha-\alpha$ systems. The background phase shift is also obtained using the residual continuum solutions in the CSM. We discuss the problems of scattering in this framework, and show that this method is very useful in the investigation of the effect of the resonance in the observed scattering cross sections.

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2. Theoretical framework

2.1. Complex scaling method

The CSM has been introduced to determine resonant states within L^2 basis functions, and is defined by the following complex-dilatation transformation for relative coordinate $\vec{r}$ and momentum $\vec{k}$

$$\vec{r} \to \vec{r} e^{i\theta}, \quad \vec{k} \to \vec{k} e^{i\theta} \tag{1}$$

where θ is a scaling angle and $0 < \theta < \theta_{\max}$. The maximum value $\theta_{\max}$is determined to keep analyticity of the potential. For example, $\theta_{\max} = \pi/4$ for a Gaussian potential. This transformation makes every branch cut to rotate by -2θ on the complex energy plane. Applying this transformation, we can write the complex-scaled Schrödinger equation as follows:

$$H^{\theta}\Psi^{\nu}_{J^{\pi}}(\theta) = E^{\theta}_{\nu}\Psi^{\nu}_{J^{\pi}}(\theta) \tag{2}$$

The complex-scaled Hamiltonian H^{θ} and wave function $\Psi^{\nu}_{J^{\pi}}(\theta)$ are defined as $U(\theta)\,HU(\theta)^{-1}$ and $U(\theta)\Psi^{\nu}_{J^{\pi}}$, respectively—see [1,2] for details.

Applying the L^2 basis function method, the radial wave function is expanded as

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$$\Psi^{\nu}_{J^{\pi}}(\theta)=\sum_{i=1}^{N}c_i^{J^{\pi}\nu}(\theta)\phi_i(r) \tag{3}$$

where $\phi_i(r)$ is an appropriate basis function set. The expansion coefficients $c_i^{J^{\pi}\nu}$ and the complex energy eigenvalues E_ν^θ are obtained by solving the complex eigenvalue problem given in Eq. (2). The complex energies of the resonant states are obtained as $E_r=E_r^{res}-i\Gamma/2$, when $\tan^{-1}(\Gamma_r/2E_r^{res})<2\theta$.

To solve the eigenvalue problem of Eq. (2), we employ the Gaussian basis functions given as follows:

$$\phi_i(r)=N_l(b_i)r^l\exp\left(-\frac{1}{2b_i^2}r^2\right)Y_{lm}(\hat{r}), \tag{4}$$

where the range parameters are given by a geometric progression as $b_i=b_0\gamma^{i-1}$; $i=1,\ldots,N$, and $N_l(b_i)$ is the normalization factor. We take $N=60$ and employ the optimal values of b_0 and γ to obtain stationary solutions. All results are obtained with $\theta=15°$.

2.2. Continuum-level density and phase shift

The CLD $\Delta(E)$ is given as

$$\Delta(E)=-\frac{1}{\pi}\mathrm{Im}\left\{\mathrm{Tr}\left[G^+(E)-G_0^+(E)\right]\right\} \tag{5}$$

where

$G^+(E)=(E+i\varepsilon-H)^{-1}$ and

$$G_0^+(E)=(E+i\varepsilon-H_0)^{-1}$$

are the full and free Green's functions, respectively. In this study, the Hamiltonian H and H_0 are transformed using the CSM.

The CLD is related to the scattering phase shift $\delta(E)$; it can be expressed in the following form in the single channel case [7]:

$$\Delta(E)=\frac{1}{\pi}\frac{d\delta(E)}{dE} \tag{6}$$

Using this relation, we can obtain the phase shift as a function of the eigenvalues in the complex scaled Hamiltonian by integrating the CLD.

When we expand the wave functions in terms of the finite number of basis states N, the discretized eigenstates are obtained with number N and the level density can be approximated as in [7]:

$$\Delta(E)\approx\Delta_\theta^N(E)=-\frac{1}{\pi}\mathrm{Im}\left[\sum_{b=1}^{N_b}\frac{1}{E+i0-E_b}+\sum_{r=1}^{N_r^\theta}\frac{1}{E-E_r^{res}+i\Gamma_r/2}+\sum_{c=1}^{N_c^\theta}\frac{1}{E-\varepsilon_c^r+i\varepsilon_c^i}-\sum_{k=1}^{N}\frac{1}{E-\varepsilon_k^{0r}+i\varepsilon_k^{0i}}\right] \tag{7}$$

where $N=N_b+N_r^\theta+N_c^\theta$ is the total number of N_b (bound states), N_r^θ (resonance states), and N_c^θ (continuum states) solutions. Then, we can obtain the phase shift

$$\delta_\theta^N(E)=N_b\pi+\sum_{r=1}^{N_r^\theta}\left\{-\cot^{-1}\left(\frac{E-E_r^{res}}{\Gamma_r/2}\right)\right\}+\sum_{c=1}^{N_c^\theta}\left\{-\cot^{-1}\left(\frac{E-\varepsilon_c^r}{\varepsilon_c^i}\right)\right\}-\sum_{k=1}^{N}\left\{-\cot^{-1}\left(\frac{E-\varepsilon_k^{0r}}{\varepsilon_k^{0i}}\right)\right\} \tag{8}$$

where $E\geq 0$. When we define δ_r, δ_c, and δ_k as

$$\cot\delta_r=\frac{E_r^{res}-E}{\Gamma_r/2},\quad \cot\delta_c=\frac{\varepsilon_c^r-E}{\varepsilon_c^i},\quad \cot\delta_k=\frac{\varepsilon_k^{0r}-E}{\varepsilon_k^{0i}} \tag{9}$$

respectively, we can write the phase shift as

$$\delta_\theta^N(E)=N_b\pi+\sum_{r=1}^{N_r^\theta}\delta_r+\sum_{c=1}^{N_c^\theta}\delta_c-\sum_{k=1}^{N}\delta_k \tag{10}$$

The geometrical indications for δ_r, δ_c, and δ_k are given for two energy cases, larger or smaller than the real parts of the eigenenergies E_r, ε_c, and ε_k, as shown in Fig. 1. The phase shift δ_r for the resonances is the angle of the r th resonant pole measured at the energy E on the real energy axis. At $E=E_r^{res}$, we have $\delta_r=\pi/2$ for every resonant pole. In addition, $\delta_r=\tan^{-1}(\Gamma_r/2E_r^{res})>0$ at $E=0$ and $\delta_r=\pi$ at $E=\infty$ for each resonance. Similarly, phase shifts from continuum terms including the asymptotic part, δ_k, are given by the angles of the discretized continuum energies. At $E=\infty$, the continuum terms of the phase shifts go to $-(N_b+N_r^\theta)\pi$ because of the relation $N=N_b+N_r^\theta+N_c^\theta$.

2.3. Cross section

The cross section is described using the calculated phase shifts; we can identify the contributions from every resonant pole and continuum term. When we concentrate our interest on the contribution from a single resonant pole and other terms that are mainly described as background phase shift, we can achieve the same results as those of Fano [8]. The total and partial reaction cross sections can be calculated using the results of the phase shifts

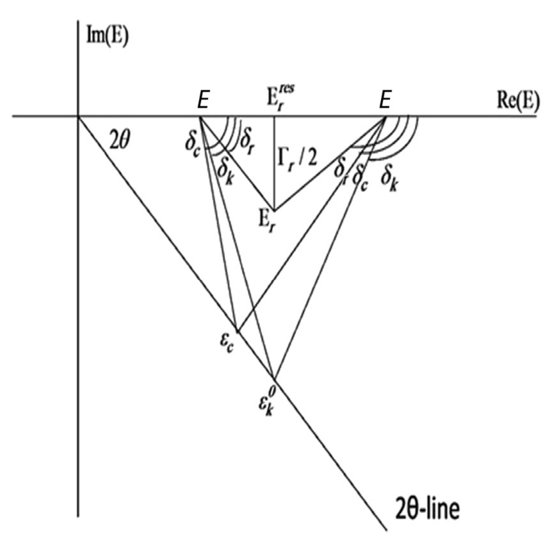

Fig. 1. Geometrical indications of phase shifts: δ_r, δ_c, and δ_k.