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Response of the point-reactor telegraph kinetics to time varying reactivities

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A B S T R A C T

The new model of the Point Reactor Kinetics (PRK) equations developed based on the Telegraph approximation of the neutron transport equation, is solved for several cases of time varying Reactivities insertions and Temperature feedback while comparing it to that of the diffusion PRK model in an infinite Thermal Homogenous Nuclear Reactor. Diffusion PRK is based on the Neutron Diffusion Equation which is a parabolic differential equation and hence it assumes an infinite velocity of propagation, while neutrons propagate with a finite velocity. By the introduction of the hyperbolic type Telegraph equation which is a more accurate representation of the neutron transport than the diffusion equation and in which neutrons propagate with a finite velocity, one could overcome this paradox that contradicts causality. The new model introduces a new parameter called the relaxation time (τ), which is not present in the diffusion approximation, and affects the neutron density calculations. Both Ramp insertions of reactivity and Sinusoidal insertions of reactivity were studied, as well as the effect of The Adiabatic Temperature feedback. The general phenomena in the solution of the new model is a Relaxation in the time response of the solution. It is found that the Telegraph model with its extra second order time derivative, will give observable different values than that of the diffusion even when we used small (τ) especially for the cases at which the neutron density changes rapidly.

1. Introduction

The system of the neutron point reactor kinetics (PRK) equations is one of the most important reduced models of nuclear engineering. They have been the subject of countless studies and applications to understand the neutron dynamics and its effects and to solve it using several methods as well as expand it to nonlinear and two energy group formulations (Nahla, 2015; Aboanber et al., 2014; Sérgio et al., 2014; Petersen et al., 2011; Nahla and Zayed, 2010) and reflected reactors (Aboanber, 2009; Aboanber and El Mhlawy, 2009). However, every PRK study was based on the neutron diffusion equation, which is considered as the correct first order approximation of the neutron transport equation. The Neutron Diffusion Equation is a differential equation of a parabolic type, and hence the neutrons described by it carry an infinite propagation velocity which in turns contradict causality principle. This can be found in details in several publications, for e.g. Weinberg and Noderer (1951), Weinberg and Wigner (1958), Meghreblian and Holmes (1960) and Beckurts and Wirtz (1964). It is also found in several recent publications, for e.g. Altahhan et al. (2016), Espinosa-Paredes and Polo-Labarrios, (2012), Heizler (2010), Olson et al. (2000), Das (1998) and Masoliver and Weiss (1994).

Another approximation to the time dependent neutron transport equation has been considered where it overcomes this infinite velocity effect. This approximation is identified as the neutron telegrapher's equation, the neutron telegraph equation or the neutron telegraphist's equation (Weinberg and Noderer, 1951; Weinberg and Wigner, 1958; Meghreblian and Holmes, 1960; Beckurts and Wirtz, 1964; Altahhan et al., 2016), and its derivation is based on the neutron transport equation and is presented in

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references Weinberg and Noderer (1951), Meghreblian and Holmes (1960) and Beckurts and Wirtz (1964). Starting from the mono-energetic transport equation in the flux form and using the P_N approximation method, the P_1 approximation of the neutron Transport Equation can be attained and is given by a system of two coupled partial differential equations:

$$\left.\begin{aligned}\frac{3}{v}\frac{\partial \mathbf{J}\left(\vec{r},t\right)}{\partial t}+\nabla\phi\left(\vec{r},t\right)+3\Sigma_{tr}\left(\vec{r}\right)\mathbf{J}\left(\vec{r},t\right)=0\\ \nabla\cdot\mathbf{J}\left(\vec{r},t\right)+\frac{1}{v}\frac{\partial\phi\left(\vec{r},t\right)}{\partial t}+\Sigma_a\left(\vec{r}\right)\phi\left(\vec{r},t\right)=S\left(\vec{r},t\right)\end{aligned}\right\} \tag{1}$$

Here $\mathbf{J}(\vec{r},t)$ is the neutron current density vector and it depends on both the position vector $\vec{r}$ and the time t, $\phi(\vec{r},t)$ is the neutron flux, v is the neutron speed, Σ_{tr} is the transport macroscopic cross section, Σ_a is the macroscopic absorption cross-section and $S(\vec{r},t)$ is the source term. The spatial and temporal dependences will be omitted but should hold unless stated otherwise. When neglecting the derivative of the neutron current with respect to time $(\partial\mathbf{J}(\vec{r},t)/\partial t)$, the resulting partial differential equation after combination is the diffusion equation. viz. Equation (2):

$$\frac{1}{v}\frac{\partial\phi}{\partial t}=D\nabla^2\phi-\Sigma_a\phi+S \tag{2}$$

where D is the diffusion coefficient. On the other hand if the neglected term is retained, thus retaining the full form of the P_1 approximation, the neutron telegraph equation is obtained when combining the above system of two equations to a single one. That is, the neutron telegraph equation is the first order approximation of the P_N approximation of the transport equation, viz. Equation (3):

$$\frac{3D}{v^2}\frac{\partial^2\phi}{\partial t^2}+\frac{(1+3D\Sigma_a)}{v}\frac{\partial\phi}{\partial t}=D\nabla^2\phi-\Sigma_a\phi+S+\frac{3D}{v}\frac{\partial S}{\partial t} \tag{3}$$

Considering that the above result was obtained directly from the transport equation (Weinberg and Noderer, 1951; Meghreblian and Holmes, 1960; Beckurts and Wirtz, 1964), we expect Eq. (3) to be a more accurate statement of the time dependent problem when using the P_1 approximation to the transport equation. And hence, for a first order approximation, the Telegraph equation with its additional derivatives terms and variables, is a more accurate representation of the neutron Transport than the diffusion equation, and evidently the diffusion theory implies an additional assumption of $(\partial\mathbf{J}(\vec{r},t)/\partial t)=0$ which have resulted in the omission of the additional terms in Eq. (3). Since the neutron current density gives an indication of the flow of neutrons from one point to another in the same direction inside a medium, then neglecting its derivative with respect to time is equivalent to assuming that the neutrons instantaneously flow from a point to another, without taking into account the time required physically for the flow.

According to several authors (Weinberg and Noderer, 1951; Weinberg and Wigner, 1958; Meghreblian and Holmes, 1960; Beckurts and Wirtz, 1964; Altahhan et al., 2016; Espinosa-Paredes and Polo-Labarrios, 2012; Heizler, 2010; Olson et al., 2000), The neutrons propagation speed is finite and equal to $v/\sqrt{3}$ in the telegraph equation. This is a consequence of the approximate character of the angular distribution of the P_1 approximation (Weinberg and Noderer, 1951; Weinberg and Wigner, 1958). But, it is better than the infinite propagation velocity of the neutron diffusion equation. Recently, a few revisions were made to this value prompting a changed Telegraph equation, for instance (Espinosa-Paredes and Polo-Labarrios, 2012; Heizler, 2010; Olson et al., 2000). It must be emphasized that the analysis done in (Espinosa-Paredes and Polo-Labarrios, 2012; Heizler, 2010; Olson et al., 2000), does not include a model of PRK.

Several models of Point Reactor Kinetics are found in the literature that is seemingly based on the P_1 approximation (Espinosa-Paredes et al., 2011; Nunes et al., 2015; Niederauer, 1967). They are different from our model in that they are either a fractional Telegraph equation model (G. Espinosa-Paredes et al. (2011), or the methodology they used to arrive at their model is totally different from the telegraph equation as well as the results following their analysis, according to the authors in Nunes et al. (2015), or is consistent with our model, although with different variables (Niederauer (1967). It is to be noted that the model in (Espinosa-Paredes et al., 2011) has been the inspiration of our Non-Fractional order telegraph equation model found in Altahhan et al. (2016) and in this paper. Very Detailed comparison between those different models is found in (Altahhan et al., 2016).

In this paper, the telegraph PRK model of (Altahhan et al., 2016) has been solved for different cases of time varying insertions of reactivity and Temperature feedback, and the solutions compared to those of the diffusion model for the same insertions for an infinite thermal homogenous nuclear reactor. In Section 2, a brief introduction of the model is found as well as an explanation is presented regarding the numerical method adopted to solve the new model. While in section 3 we solve the new model for a ramp insertion of reactivity case. Section 4 holds the results of the sinusoidal insertion of reactivity. In section 5 the case of a Temperature feedback is reported, while in section 6 a concluding discussion is given regarding the results.

2. Mathematical model and the method of solution

Altahhan et al. (2016) formulated a point reactor kinetics model in light of the neutron Telegraph Equation, called the (TPRK) model, and solved it for a step insertion of reactivity as well as solving analytically the model for different known approximations in Reactor kinetics field like the prompt jump approximation and the constant delayed neutrons approximation, while also deriving the matrix form of the model necessarily for numerical solutions of the model for time varying reactivities insertions and temperature feedback. The PRK equations for an infinite homogenous reactor and m delayed neutrons groups with no external source based on the telegraph equation are given by Altahhan et al. (2016):

$$\tau\frac{d^2n(t)}{dt^2}+\left[1-\tau\left(\frac{\rho(t)-\beta}{\Lambda}\right)\right]\frac{dn(t)}{dt}=\frac{\rho(t)-\beta}{\Lambda}n(t)+\sum_{i=1}^{m}\lambda_iC_i(t)+\tau\sum_{i=1}^{m}\lambda_i\frac{dC_i(t)}{dt},\quad \frac{dC_i(t)}{dt}=\frac{\beta_i}{\Lambda}n(t)-\lambda C_i(t) \tag{4}$$

With initial conditions given by:

$$n(0)=n_0,\quad \left.\frac{dn}{dt}\right|_{t=0}=0,\quad C(0)=C_0=\frac{\beta}{\Lambda\lambda}n_0 \tag{5}$$

where $n(t)$ is the neutron density that depends on time t, τ is the relaxation time of an infinite reactor, $\rho(t)$ is the reactivity as a function of time, β is the total delayed neutron fraction of the fission neutrons while β_i is the delayed neutron fraction of the fission neutrons for the i^{th} delayed neutrons precursor group, Λ is the prompt neutrons mean generation time, m is the number of delayed neutrons precursor groups, λ_i is the delayed neutron decay constant for the i^{th} delayed neutrons precursor group and $C_i(t)$ is the neutrons precursor concentration in the i^{th} group. The