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Solution of the fractional neutron point kinetics equations considering time derivative of the reactivity

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ABSTRACT

In this paper, we present a numerical solution for the time fractional neutron point kinetics equations (FNPK) model with Newtonian temperature feedback effects on finite medium. We consider reformulation of the FNPK model to include the time derivative of the reactivity. The model of FNPK describes a finite bare reactor core associated with values of the nuclear parameters which are adjusted to preserve the criticality of the reactor at the steady state. The modified model is compared with the earlier published models and we have found that they are consistent when the reactivity is constant. Implicit difference method is proposed to solve the model of the FNPK with six groups of delayed neutron precursors. The method is based on approximating the Caputo fractional derivative by shifted Grünwald-Letnikov fractional derivative. Stability of the method is discussed and it has proved that the method is unconditionally stable. The neutron density results with temperature reactivity feedback, for different values of fractional orders ($0 < \alpha \leq 1$), are shown and compared with the classical neutron point kinetics equations.

1. Introduction

The fraction calculus generalizes the integration and differentiation with ordinary- (integer-) order to be of non-integer order. There are many of dynamical systems of engineering, physics, chemistry that can be modelled by fractional calculus, e.g., electrochemical process (Sun et al., 1984), viscoelastic materials (Bagley and Calico, 1991; Koeller, 1984), bioengineering (Magin, 2006), solid mechanics (Rossikhin and Shitikova, 1997), bioelectrode independence (Magin, 2006) and fractional kinetics (Kusnezov et al., 1999; Zaslavsky, 2002).

The fractional neutron point kinetics (FNPK) model generalizes the classical neutron point kinetics (CNPK) model with integer derivative to be of non-integer derivative, Espinosa-Paredes et al. (2011). These models constitute a useful tool to provide important information on the dynamics of the reactor. Espinosa-Paredes and Labarrios (2016) studied the effects of each term of the fractional neutron point kinetics (FNPK) equations on the neutron density, the concentration of the neutron precursors and the source term. Also, Espinosa-Paredes (2016a) derived a new model for the fractional space neutron point kinetics (F-SNPK) equations for finite medium based on the fractional-space law for the neutron density current. The CNPK equations with Newtonian temperature feedback are solved by many authors (for examples: Hamada, 2013; Ganapol, 2013; Picca et al., 2013) while the FNPK equations with temperature feedback are presented and discussed by Espinosa-Paredes et al. (2014), Patra and Saha (2015), Nowak et al. (2015) and Schramm et al. (2016).

The cross sections that appeared in the one speed diffusion model are averages of the true energy dependent microscopic cross sections over an energy spectrum. So, the average values of these cross sections should be adjusted carefully according to the reactor operating condition, i.e., the critical conditions at the steady state. The multiplication factor and hence the reactivity $\rho(t) = [k(t) - 1]/k(t)$ appearing in the CNPK or FNPK systems is a given function of time. The multiplication factor depends on the size and composition of the reactor according to a specific relation derived for the one speed diffusion model of a bare reactor core. This relation relates the size and the composition through the geometric buckling and the macroscopic cross sections. The macroscopic cross sections themselves involve the atomic number densities of the materials in the core. Therefore, any changes of the macroscopic cross sections due to the temperature or the control rod movements (power level) will influence the multiplication

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factor and hence the reactivity (Duderstadt and Hamilton, 1976). Thus, the reactivity feedback plays an important role in the reactor operation and it is treated as a nonlinear function of the power level or the temperature.

In this paper, we have re-derived the model of the FNPK equations obtained by Espinosa-Paredes et al (2011) introducing a new term for the reactivity derivative. The model describes the one speed diffusion model for finite (or infinite) bare reactor core. Average values of the nuclear parameters are estimated according to the critical conditions of the reactor at the steady state. The geometric buckling and the non-leakage probability terms are considered during the derivation process. On the other hand, based on an approximation of the Caputo derivative, shifted Grünwald-Letnikov formula is used to introduce an efficient implicit difference approximation. We have applied the proposed method to solve the modified model with six groups of delayed neutron precursors. Stability of the proposed method is studied; it has proved that the method is unconditionally stable. Numerical evaluation performed by the considered scheme has been coded by MATLAB 8 for personal computer.

2. Preliminaries

For $t > t_0$ and $m-1 < \alpha \le m \in Z^{+}$, we can define the following (Podlubny, 1999):

Definition 1. The Caputo fractional derivative of $f(t)$ of order $\alpha > 0$, $t_0 \ge 0$ is:

$$ {}_{t_0}^{C}D_t^{\alpha} f(t) = \frac{1}{\Gamma(m-\alpha)} \int_{t_0}^{t} (t-\tau)^{m-\alpha-1} \frac{d^m}{d\tau^m} f(\tau) d\tau, \tag{1} $$

Definition 2. The standard Grünwald-Letnikov formula for the fractional derivative of $f(t)$ is:

$$ {}_{t_0}^{GL}D_t^{\alpha} f(t) = \lim_{N\to\infty} \left\{ \frac{\left(\frac{t-t_0}{N}\right)^{-\alpha}}{\Gamma(-\alpha)} \sum_{j=0}^{N} \frac{\Gamma(j-\alpha)}{\Gamma(j+1)} f\left(t - j\left(\frac{t-t_0}{N}\right)\right) \right\}, \tag{2} $$

If $h_N = \left(\frac{t-t_0}{N}\right)$ and $\omega_j^{(\alpha)} = \frac{\Gamma(j-\alpha)}{\Gamma(-\alpha)\Gamma(j+1)} = (-1)^j \binom{\alpha}{j}$, *then* Eq. (2) *turns to*

$$ {}_{t_0}^{GL}D_t^{\alpha} f(t) = \lim_{N\to\infty} \left\{ h_N^{-\alpha} \sum_{j=0}^{N} \omega_j^{(\alpha)} f(t - jh_N) \right\} \cong h_N^{-\alpha} \times \sum_{j=0}^{N} \omega_j^{(\alpha)} f(t - jh_N) + O(h) \tag{3} $$

If $\alpha = 1$, we have ${}_{t_0}^{GL}D_t^{1} f(t_k) = \frac{f(t_k) - f(t_{k-1})}{h}$. Clearly, the weights ω_j^{α} depend on the fractional order α and the index j. For example: $\omega_0^{\alpha} = 1$, $\omega_1^{\alpha} = -\alpha$, $\omega_2^{\alpha} = \frac{\alpha(\alpha-1)}{2!}$, $\omega_3^{\alpha} = \frac{-\alpha(\alpha-1)(\alpha-2)}{3!}$, ….

Definition 3. The shifted Grünwald-Letnikov formula is (Meerschaert and Tadjeran, 2006):

$$ \frac{d^{\alpha} f(t_k)}{dt^{\alpha}} = \frac{1}{h^{\alpha}} \sum_{j=0}^{k} \omega_j^{\alpha} f\left(t_{k-j+1}\right) + O(h) \tag{4} $$

Definition 4. Leibniz rule

Suppose that $\varphi(t)$ and $f(t)$ along with all their derivatives are continuous in $[0, t]$, then the fractional order derivative of the product is (Podlubny, 1999):

$$ {}_0D_t^{\alpha}[\varphi(t) f(t)] = \sum_{k=0}^{\infty} \binom{\alpha}{k} \varphi^{(k)}(t)\, {}_0D_t^{\alpha-k} f(t) \tag{5} $$

Definition 5. For the constants λ and μ, the fractional differentiation is linear

$$ D^{\alpha}[\lambda f(t) + \mu g(t)] = \lambda D^{\alpha} f(t) + \mu D^{\alpha} g(t) \tag{6} $$

3. Modified fractional neutron point kinetics equations

The one speed diffusion model of the nuclear reactor can be described by the following equations (Duderstadt and Hamilton, 1976):

$$ \frac{1}{v} \frac{\partial \varphi(\boldsymbol{r}, t)}{\partial t} + \nabla \boldsymbol{J}(\boldsymbol{r}, t) + \Sigma_a \varphi(\boldsymbol{r}, t) = S(\boldsymbol{r}, t), \tag{7} $$

$$ \frac{1}{v} \frac{\partial \boldsymbol{J}(\boldsymbol{r}, t)}{\partial t} + \Sigma_{tr} \boldsymbol{J}(\boldsymbol{r}, t) + \frac{1}{3} \nabla \varphi(\boldsymbol{r}, t) = S_1(\boldsymbol{r}, t), \tag{8} $$

and the neutron source $S(\boldsymbol{r}, t)$ is defined as:

$$ S(\boldsymbol{r}, t) = (1-\beta)\nu \Sigma_f \varphi(\boldsymbol{r}, t) + \sum_{i=1}^{G} \lambda_i C_i(\boldsymbol{r}, t), \tag{9} $$

and $S_1(\boldsymbol{r}, t) = 0$ is the first moment of the neutron source, $(cm^{-3}s^{-1})$. C_i is the precursor concentrations $(atom/cm^3)$ of the i th group of delayed neutrons that satisfy the balance equation:

$$ \frac{\partial C_i(\boldsymbol{r}, t)}{\partial t} = -\lambda_i C_i(\boldsymbol{r}, t) + \beta_i \nu \Sigma_f \varphi(\boldsymbol{r}, t),\ i = 1, 2, \ldots, G, \tag{10} $$

where

$\varphi(\boldsymbol{r}, t)$:is the neutron flux at position $\boldsymbol{r}$ and time t $(cm^{-2}s^{-1})$,
$\boldsymbol{J}(\boldsymbol{r}, t)$:is the neutron current density $(cm^{-2}s^{-1})$,
Σ_a:is the macroscopic absorption cross section (cm^{-1}),
D:is the diffusion coefficient (cm),
β_i:is the fraction of i th group of delayed neutrons, $(i = 1, 2, \ldots, G)$,
β:is the total fraction of delayed neutrons,
λ_i:is the decay constant of i th group of delayed neutron precursor (s^{-1}),
ν:is the mean number of fission neutrons,
Σ_f:is the macroscopic fission cross section (cm^{-1}),
v:is the neutron velocity (cm/s).

We consider here one speed diffusion model where the neutrons are diffusing in a homogenous medium and hence the quantities $D = \frac{1}{3\Sigma_{tr}}$, $\Sigma_a = \Sigma_t - \Sigma_s$ and $\Sigma_{tr} = \Sigma_t - \overline{\mu}_0 \Sigma_s$ (where $\overline{\mu}_0$ is the average cosine of the scattering angle in the neutron scattering collision) don't depend on the position. Indeed, the two functions $\varphi(\boldsymbol{r}, t)$ and $\boldsymbol{J}(\boldsymbol{r}, t)$ are related by Fick's law (Duderstadt and Hamilton, 1976) by the relation $\boldsymbol{J}(\boldsymbol{r}, t) = -D\nabla\varphi(\boldsymbol{r}, t)$. Eqs. (7) (8) and (9) represent P_1 approximation. Let us divide Eq. (8) by Σ_{tr} and take the fractional derivative on the first term (Espinosa-Paredes et al., 2008) to get a generalization of Eq. (8) of the form: