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Study on random walk and its application to solution of heat conduction equation by Monte Carlo method

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ABSTRACT

In this paper optimized and fast numerical method based on Monte Carlo simulation are introduced to achieve several algorithms for solving the steady state and transient heat conduction problems with any prescribed convection boundary conditions and interior thermal source distribution. The methods are mesh-less and thus can solve problems with complex geometries, non-uniform heat source or porous bodies with no inherent limitation. In the proposed approach the temperature of any point in the studying body can be found independently from other points without solution of related system of linear algebraic equations. As part of this study, an effort was undertaken to summarize various random walk methods in Monte Carlo. The Monte Carlo methods that are introduced in this paper use Semi Floating Random Walk (SFRW) and Full Floating Random Walk (FFRW) to estimate temperature in the domain of the definition. Also, Fixed Random Walk (FRW) method is applied to solve transient heat conduction equation. Each method has its own unique features that will be discussed during this work. It was observed that FFRW method is the fastest method to steady state conditions and also FRW method has capability to run both steady state and transient heat conduction equation. It was showed that the results are in good agreement with those of the finite difference method.

1. Introduction

Monte Carlo has been defined as the technique of solving a problem by putting in random numbers and getting out random answers (Howell, 1964). This technique is basically any technique making use of random numbers to solve a problem. Problems handled by the Monte Carlo method are of two types. The first is called probabilistic and the second type is called deterministic, the naming depending on the behavior and outcome of random processes. Whether or not the Monte Carlo method can be applied to a given problem doesn't depend on the stochastic nature of the system being studied, but only on the investigator's ability to formulate the problem in such a way that random numbers may be used to obtain the solution (Oluwajobi and Jeje, 2008). This method was first developed at Los Alamos during the WWII (World War II) Manhattan Project for the purposes of modeling neutron trajectories during fission. Since that time, the Monte Carlo method has undergone numerous developments and has enjoyed applications in virtually every area of science and engineering. This method have been applied successfully for solving differential and integral equations, for finding eigenvalues, for inverting matrices, and for evaluating multiple integrals (Price and Story, 1985; Sabelfeld, 1991; Sobol, 1994). It had been known since early last century that probability-sampling (Monte Carlo) techniques could be employed in solving partial differential equations. A valuable survey of Monte Carlo methods as applied to the solution of differential and difference equations has been provided by Curtiss (1950). A prime reference source for the random walk technique, which is basic to Monte Carlo methods, is Feller's text on probability theory (Feller, 1964). Monte Carlo techniques for solving heat conduction problems have been reported by (Haji-Sheikh and Sparrow, 1966; Hoffman, 1976; Hoffman and Bands, 1975) and a summary of this method is given in (Hoffman, 1976). Continues works such (Fraley et al., 1980; Haji-Sheikh and Sparrow, 1967; Hoffman and Bands, 1976) have suggested that the Monte Carlo approach can also be useful in the solution of certain classes of heat conduction problems. Heat conduction in solids is one of the most important topics in heat transfer because of its numerous applications in various branches of science and engineering. Kowsary, and Arabi (Kowsary and Arabi, 2000) introduced a method to

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Nomenclature

T	Temperature
T_{wall}	Wall temperature
$\boldsymbol{T}_{\infty}$	Ambient temperature
$q^{\cdot}$	Thermal source
x	Horizontal direction
y	Vertical direction
Δx	Increment in x
Δy	Increment in y
ρ	density
P_{x^+}	Probability of stepping to the point $(x + \Delta x, y)$
P_{x^-}	Probability of stepping to the point $(x - \varDelta x, y)$
P_{y^+}	Probability of stepping to the point $(x, y + \varDelta y)$
P_{y^-}	Probability of stepping to the point $(x, y - \varDelta y)$
e	Random number
N	Total number of random walk
h	Heat transfer coefficient
k	Thermal conductivity
c_p	Specific heat capacity

solution of anisotropic heat conduction based on the fixed-step random walk procedure using Monte Carlo algorithm. They obtained steady-state temperature distribution by the Monte Carlo calculations for a two-dimensional anisotropic solid having simple geometry and boundary conditions.

The traditional numerical methods for heat conduction problems such as the finite difference, finite element and finite volume are well developed. However, these methods are based on discretized mesh systems, thus they are inherently depend on problem geometry and applying of these methods for complex geometries in some cases are difficult. Also, Sometimes, in practical conditions determining of an arbitrarily point (for example the points at which maximum temperature is occurred) is required, while in usual numerical methods, all mesh points temperatures must be determined simultaneously. In this paper a new approach based on Monte Carlo method for solution of heat conduction problems is developed. This method, which uses random numbers, will allow the solution of problems with as much geometric complexity. This method is able to obtain the temperature of any isolated point in solids without considering the other existing mesh points. Hence this method can be apply to find the temperature at the important points in the complex geometries (for example for the point at which melting may be occurred). In the presented method the discretization is not required and also irrespective of the problem geometry, boundary conditions and location of the unknown point; temperature of any arbitrary point can be found with desired accuracy.

The Monte Carlo method can also be used in the solution of transient heat conduction equation. But in this case, according to the governing equations, we are faced with a new conditions. A typical point temperature at new time is depends on the same point temperature at old time and adjacent points at new time. Therefore, in this approach we required to track all mesh points temperature and a lot of time are consumed for this purpose.

Most of the earlier Monte Carlo methods for heat conduction are based on discretized mesh systems and most of them are related to problems with constant temperature boundary. In the present work a Comprehensive method based on Monte Carlo calculation is introduced and solution algorithms are described in detail. With this method, we can solve any problem with any complexity and boundary condition. Rest of paper is organized as follows:

In section 2, we are going to present simulation methodology and classification of random walk method. This section has divided in four subsection and in each subsection a brief description about method and its formulation are prescribed. In section 3, different model for boundary condition are investigated. In next section different kind of boundary condition presented and its subsection formulated them for implementation in programming language. In last section ability of methods for five typical problems will be analyzed.

2. Simulation methodology

Perhaps the most straight-forward application of probability for differential equations solution can be seen in the random walk model. Random walk is similar to Brownian motion of a particle in a fluid but in scaling limit. First step in random walk model is random number generation. True random numbers are generated in non-deterministic way, not-predictable and they are not repeatable. Such sequence can only be generated by one of several sources of randomness like: keyboard timing, electrical noise from a resistor or semiconductor, radio channel or audible noise and decay times of radioactive materials. Many of the deficiencies of currently standard Monte Carlo methods for such differential equations may be traced to the relatively inflexible nature of the random-walk pattern, whereby the length of the step and the positions of the mesh points are preselected. A random walk of this type is designated here as a fixed random walk. However in Monte Carlo studies, one often uses the word ‘random’ with a slightly different meaning. It is usually applied to sequences of which once they have been determined, are not at all random in the statistical sense, but may have some properties, which are similar to the properties of a random sequence. They are sometimes called pseudo random numbers (Oluwajobi and Jeje, 2008). Generally speaking there are three types of random walks.

1) Fixed Random Walk: Particle step size is fixed and pre-assigned and also the direction of particle motion are limited in pre-assigned directions.
2) Semi Floating Random walk: Particle Step size is fixed, but the direction of particle motion is not limited. This means moving with all angles is possible and it is determined randomly.
3) Full Floating Random Walk: In this case the step size is not pre-assigned and changes at each step and the direction of particle motion is not limited, this means moving with all angles is possible and is determined randomly.

The fixed walk technique has long computational times and has difficulties with complex boundary conditions at irregular boundaries. But this method is capable for handling the transient problems. On the other hand, two other procedures are relatively independent to the shape of the boundary or the boundary conditions, and the computational time is substantially less. In each of the above cases, when the particle reaches to the specified temperature, its moving is terminated; in other words the particle is absorbed. When the particle absorbed, temperature of corresponding point is tallied.

The Monte Carlo method which will be discussed in this study find the temperature of any point independently based on the random walk of a hypothetical particle in the solid body, whereas the others numerical methods have equations for all considered nodes which one must solve them simultaneously. In continue we are going to illustrate concept of random walks and formalized them for implementation.