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Application of high-order numerical schemes and Newton-Krylov method to two-phase drift-flux model

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A B S T R A C T

This paper concerns the application and solver robustness of the Newton-Krylov method in solving two-phase flow drift-flux model problems using high-order numerical schemes. In our previous studies, the Newton-Krylov method has been proven as a promising solver for two-phase flow drift-flux model problems. However, these studies were limited to use first-order numerical schemes only. Moreover, the previous approach to treating the drift-flux closure correlations was later revealed to cause deteriorated solver convergence performance, when the mesh was highly refined, and also when higher-order numerical schemes were employed. In this study, a second-order spatial discretization scheme that has been tested with two-fluid two-phase flow model was extended to solve drift-flux model problems. In order to improve solver robustness, and therefore efficiency, a new approach was proposed to treating the mean drift velocity of the gas phase as a primary nonlinear variable to the equation system. With this new approach, significant improvement in solver robustness was achieved. With highly refined mesh, the proposed treatment along with the Newton-Krylov solver were extensively tested with two-phase flow problems that cover a wide range of thermal-hydraulics conditions. Satisfactory convergence performances were observed for all test cases. Numerical verification was then performed in the form of mesh convergence studies, from which expected orders of accuracy were obtained for both the first-order and the second-order spatial discretization schemes. At last, the drift-flux model, along with numerical methods presented, were validated with three sets of flow boiling experiments that cover different flow channel geometries (round tube, rectangular tube, and rod bundle), and a wide range of test conditions (pressure, mass flux, wall heat flux, inlet subcooling and outlet void fraction).

1. Introduction

Two-phase flow is an important phenomenon that is widely seen in many engineering applications. In nuclear reactor thermal-hydraulics, accurate modeling and simulation of two-phase flows are critical to the design and safety analysis of nuclear reactors. In the past, many two-phase flow models have been developed, such as homogeneous equilibrium model, drift-flux model, and two-fluid model. Many existing reactor system analysis codes, e.g., RELAP5 (USNRC, 2001) and TRACE (USNRC, 2010), employ the two-fluid two-phase flow model that treats the two phases separately with the interfacial interactions considered by closure correlations. Code implementation and numerical solving for the two-fluid model could be challenging tasks. On the other hand, the drift-flux model (Zuber and Findlay, 1965; Ishii, 1977; Ishii and Hibiki, 2011) is formulated to treat the two phases as a mixture. Although the drift-flux model has limitations in certain applications, they are still widely used in many applications due to their simplicity and applicability to a wide range of two-phase flow problems. For example, the RETRAN-3D (EPRI, 1998) code employs the drift-flux model and has many applications in reactor transient analyses. The drift-flux model has also been widely seen in many other applications, e.g., subchannel analysis of reactor fuel bundles (Khan and Yi, 1985; Hashemi-Tilehnoee and Rahgoshay, 2013a,b; Chung et al., 2012, 2013; Hajizadeh et al., 2017b), BWR core simulators (Galloway, 2010), two-phase flow instabilities analyses (Nayak, 2007; Wang et al., 2011), two-phase CFD code development (Tentner et al., 2015), and two-phase flow analysis in wellbores (Shi et al., 2005).

The difficulty of numerically solving the drift-flux two-phase flow model comes from two aspects: the first one being the

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intrinsic nonlinearity presented in almost any two-phase flow models; and the second one being the strong constraint casted by the drift-flux closure correlation. Additional difficulties come from the complex, and normally discrete, two-phase flow closure correlations in describing wall heat transfer, boiling and condensation, and etc. These discrete closure correlations could negatively impact the solver robustness. Traditionally, numerical solving of the drift-flux model relied on iterative methods that are based on operator-splitting type of algorithms (Galloway, 2010; Talebi et al., 2012). These types of iterative methods sometimes suffer from unstable (oscillatory) convergence behavior (Hajizadeh et al., 2017a). In recent years, the Jacobian-free Newton-Krylov (JFNK) method has gained many successes in solving nonlinear systems in different disciplines (Knoll and Keyes, 2004). In two-phase flow simulations, Mousseau has done the pioneering work that solves two-fluid two-phase flow problems using the JFNK method (Mousseau, 2004, 2005). Following his pioneering work, Zou and coworkers have greatly extended its applications in solving flow problems interested in reactor thermal-hydraulics, including both single-phase natural circulation problem (Zou et al., 2017a) and two-phase flow problems (Zou et al., 2015, 2016a,b,c), with consideration of high-order numerical methods (Zou et al., 2015, 2016c) and, more importantly, employing realistic two-phase flow closure correlations (Zou et al., 2016a,b,c). Challenges in numerically treating the phase appearance and disappearance phenomenon in the two-fluid two-phase flow model have also been studied in Zou et al. (2016d,e). Among these papers, two were devoted to study the Newton-Krylov method in the application of solving drift-flux two-phase flow problems (Zou et al., 2016a,b). In Zou et al. (2016a), the Newton-Krylov method was successfully applied to solve the drift-flux model using Ishii's drift-flux closure correlations. This work was later improved in Zou et al. (2016b) to address several issues revealed in Zou et al. (2016a), e.g., unphysical jump in numerical results due to discrete closure correlations. It is noted that, first-order numerical schemes were used in both studies.

This study is an extension to the two papers discussed above. There are two main objectives in this study. The first objective is to introduce second-order numerical schemes in solving the drift-flux two-phase flow problems. Although second-order numerical schemes have been successfully applied to solve both single-phase flow (Zou et al., 2017a) and two-fluid two-phase flow problems (Zou et al., 2015, 2016c), it is not so straightforward to extend them to the drift-flux model. This is due to the additional drift-flux related terms appearing in the drift-flux model. The second objective is to improve the robustness and thus efficiency of the Newton-Krylov method to solve the drift-flux two-phase flow problems. Due to the circular dependency (a concept borrowed from software engineering, and more discussions on this later) between closure correlation parameters and state variables in the drift-flux model, a nested iteration process was required inside each Newton's nonlinear iteration (Zou et al., 2016b). Such an approach caused unsatisfactory solver convergence performance, especially when it was experimented with high-order numerical methods and/or meshes were refined, although such an issue could normally be resolved by, albeit undesirably, reducing the time step size. This paper is organized as the following: Section 2 provides a detailed description on the drift-flux two-phase flow model and the related closure correlations. Section 3 is focused on numerical methods, including details on the second-order spatial discretization scheme and the Newton-Krylov method. Rigorous numerical verification in the form of mesh refinement and validation with experimental data are provided in Section 4. Further discussions and conclusions are presented in Section 5.

2. Model description

2.1. One-dimensional four-equation drift-flux model

The one-dimensional four-equation drift-flux two-phase flow model includes a two-phase mixture mass equation, a mass equation for the dispersed gas phase, and a momentum and an energy equation for the two-phase mixture,

$$\frac{\partial \rho_m}{\partial t} + \frac{\partial (\rho_m v_m)}{\partial x} = 0 \tag{1}$$

$$\frac{\partial \left(\alpha \rho_g\right)}{\partial t} + \frac{\partial \left(\alpha \rho_g v_m\right)}{\partial x} + \frac{\partial}{\partial x}\left(\frac{\alpha \rho_g \rho_f}{\rho_m}\overline{V_{gj}}\right) = \Gamma_g \tag{2}$$

$$\begin{aligned}\frac{\partial v_m}{\partial t} + v_m \frac{\partial v_m}{\partial x} = & -\frac{1}{\rho_m}\frac{\partial p}{\partial x} - g_x - \frac{f_m}{2D} v_m \left| v_m \right| \\ & -\frac{1}{\rho_m}\frac{\partial}{\partial x}\left[\frac{\alpha \rho_g \rho_f}{(1-\alpha)\rho_m}\overline{V_{gj}}^2\right]\end{aligned} \tag{3}$$

$$\begin{aligned}\frac{\partial (\rho_m e_m)}{\partial t} + \frac{\partial (\rho_m v_m h_m)}{\partial x} \quad & + \frac{\partial}{\partial x}\left[\frac{\alpha \rho_g \rho_f}{\rho_m}\Delta h_{gf}\overline{V_{gj}}\right] = q''_w a_w \\ & + \left[v_m + \frac{\alpha\left(\rho_f - \rho_g\right)}{\rho_m}\overline{V_{gj}}\right]\frac{\partial p}{\partial x}\end{aligned} \tag{4}$$

In these equations, subscripts m, g, and f denote the two-phase mixture, gas phase, and liquid phase, respectively. $\overline{V_{gj}}$ is the mean drift velocity of the gas phase. Following our previous papers (Zou et al., 2016a,b), this set of equations is re-derived from their original form (Ishii, 1977; Hibiki and Ishii, 2003; Ishii and Hibiki, 2011) for the convenience of numerical implementation. In their current forms, the mixture momentum equation was re-derived in the primitive form, and the mixture energy equation was re-derived in term of the internal energy. In addition, the stress tensor and covariance terms in the original equations were ignored.

The four primary variables to be solved from this set of four equations are p, α, v_m, and T, which are pressure, void fraction, mixture velocity, and temperature, respectively. Similar to our previous approach, there is always one of the two phases assumed to be at the saturation condition. For example, during nucleate boiling, the vapor phase is assumed to be at the saturation condition, while the liquid phase can be in the subcooled condition, such that, $T_f = T$ and $T_g = T_{sat}(p)$. Other dependent variables are calculated with water/steam properties correlations, e.g.,

$$\begin{aligned}\rho_f &= \rho_f\left(p, T_f\right) \\ \rho_g &= \rho_g\left(p, T_g\right)\end{aligned} \tag{5}$$

or by their definitions, e.g.,

$$\begin{aligned}\rho_m &= \alpha \rho_g + (1-\alpha)\rho_f \\ \rho_m e_m &= \alpha \rho_g e_g + (1-\alpha)\rho_f e_f\end{aligned} \tag{6}$$

An additional correlation (Talebi et al., 2012; Zou et al., 2016a) is provided to calculate the mean drift velocity of the gas phase, $\overline{V_{gj}}$,

$$\overline{V_{gj}} = \frac{\rho_m \langle\langle V_{gj}\rangle\rangle + \rho_m (C_0 - 1) v_m}{\rho_m - (C_0 - 1)\alpha\left(\rho_f - \rho_g\right)} \tag{7}$$

in which, C_0 is the distribution parameter, and $\langle\langle V_{gj}\rangle\rangle$ is the weighted mean drift velocity of the gas phase, both of which will be determined with closure (constitutive) models. The two phasic