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Numerical study on the Welander oscillatory natural circulation problem using high-order numerical methods

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ABSTRACT

In this paper, high-order numerical methods are investigated in a system analysis-like code. The classical Welander oscillatory natural circulation problem, which resembles a simplified example for many types of natural circulation loops widely seen in nuclear reactor systems, was chosen to illustrate the applicability of such methods in system analysis codes, and to demonstrate the advantages of such methods over the low-order methods widely used in existing system analysis codes. As originally studied by Welander, the fluid motion in a differentially heated fluid loop can exhibit stable, weakly unstable, and strongly unstable modes. A theoretical stability map has also been originally derived from the stability analysis. Numerical results obtained in this paper show very good agreement with Welander's theoretical derivations. For stable cases, numerical results from both the high-order and low-order numerical methods agree well with the non-dimensional flow rate that were analytically derived. The high-order numerical methods give much less numerical errors compared to those using low-order numerical methods. For stability analysis, the high-order numerical methods perfectly predicted the stability map even with coarse mesh and large time step, while the low-order numerical methods failed to do so unless very fine mesh and time step are used. The result obtained in this paper is a strong evidence for the benefits of using high-order numerical methods over the low-order ones, when they are applied to simulate natural circulation phenomenon that has already gained increasing interests in many existing and advanced nuclear reactor designs.

1. Introduction

Some existing and many advanced nuclear reactor designs make essential use of passive systems (IAEA, 2005, 2009, 2012). These passive systems are commonly considered to be more reliable than active systems because they rely on natural phenomena, such as natural circulation, that do not require external inputs (especially energy inputs). In many reactor designs, natural circulations are designed as the main mechanism to remove decay heat from reactors during postulated accidental scenarios, for example the direct reactor auxiliary cooling system (DRACS) used in salt-cooled high-temperature reactor designs (Varma et al., 2012; Hughes et al., 2016). In some innovative designs, natural circulations are also used to as the main core heat removal mechanism during normal operations, for example the NuScale design (Reyes, 2011). Many experimental facilities have been built to study both the single- and two-phase natural circulation phenomena, but most of them are scaled-down facilities due to the prohibitive and normally unnecessary cost to build full-scale test facilities. System analysis codes, such as RELAP5 (U.S. NRC, 2001), are widely used to analyze the scaled-down facilities to gain understanding of the natural circulation phenomenon. It is normally expected that system analysis codes can be fully validated using experimental data from the scaled-down facilities, and then they could be extrapolated in predicting the behavior of full-scaled systems, such as a nuclear reactor. However, most existing system analysis codes have been developed for light water reactor designs that in most scenarios the flow is driven by external energy (such as pumps), or by pre-stored energy (such as pressurized nitrogen in accumulators used in PWR designs). On the contrary, natural circulation relies on completely different driving mechanism, e.g., convective heat transfer and gravity force, which provide much weaker driving forces. Some phenomena key to natural circulations, such as flow instability and flow in transitional regime, pose challenges to existing codes due to lack of data and thus large model uncertainties. In addition, most

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existing system analysis codes employ first-order numerical schemes in both space and time, which produce excessive numerical diffusion not suitable for simulations of natural circulations. To active systems, it is not a critical concern because of the dominant driving force provided from active systems. To natural circulations, this excessive numerical diffusion could be critical, which can even change the qualitative trend of physical evolution of the phenomenon. In the work done by D'Auria et al. (1997), RELAP5 was used to simulate single-phase phenomenon in an experimental loop. It was concluded that, if the code was properly *tuned*, it was able to predict well for stable conditions. However, if oscillations happened, the agreement between numerical results and experimental observations were poor. For transient simulations, a similar conclusion was also drawn in the paper by Misale et al. (1999). In the work done by Mangal et al. (2012), RELAP5 is used to simulate single- and two-phase natural circulation behaviors in two experimental loops. It was found the RELAP5 results were generally good for stable conditions, while for transient conditions numerical results were very sensitive to nodalization and did not agree with experimental results. In the work done by Vijayan et al. (1995), ATHLET code was used to simulate an experimental natural circulation loop. It was found that coarse mesh always gave stable results even for physically unstable conditions. By refining the mesh, the code was able to predict oscillatory behaviors well.

The main purpose of this paper is to explore high-order numerical methods that are suitable for deployment in system analysis codes, and their applications in simulations of natural circulation phenomenon. High-order spatial schemes are widely accepted concepts in the computational fluid dynamics (CFD) community, but have rarely played their roles in the reactor thermal-hydraulics field, e.g., system analysis codes. The situation is similar to high-order implicit methods, partially because of the difficulty in solving complex non-linear systems. Previously, systematic studies were carried out by the authors to investigate high-order numerical methods and Jacobian-free Newton-Krylov (JFNK) methods as the non-linear solver in applications of single- and two-phase flow simulations (Zou et al., 2015a, b; 2016a, b, c, d, e). These combined methods were found to be promising and in this work are extended in the application of solving a classical natural circulation problem – the Welander problem (Welander, 1967). The problem was originally proposed to study the oscillatory instability of a differentially heated fluid loop. It has been extensively studied to understand the oscillatory behavior of natural circulation phenomenon (Ambrosini and Ferreri, 1998, 2003; Maiani et al., 2003; Zvirin, 1981). However, most of these studies were focusing on its instability analysis using modal methods (Ambrosini and Ferreri, 1998) or explicit time integration methods (Ambrosini and Ferreri, 1998, 2003), both of which are not suitable for the deployment in system analysis type of codes. Neither has it been thoroughly tested using system analysis type of codes. It must be emphasized that the main purpose of this paper is to investigate the applicability of high-order numerical methods in system analysis type of codes. The Welander problem by itself is an example used in this paper to illustrate such applicability and to demonstrate the advantages of the high-order numerical methods over the low-order ones that have been widely adopted in existing system analysis codes. This paper is organized as the following: the Welander problem is introduced in Section 2; single-phase flow model and numerical methods are presented in Section 3; numerical results and discussions are provided in Section 4, which is followed by conclusions in Section 5.

2. The Welander problem

The Welander oscillatory instability problem was originally proposed to study the oscillatory instability of a differentially heated fluid loop (Welander, 1967). As depicted in Fig. 1, a closed flow loop consists of four sections with a total length L and uniform pipe cross-sectional area A. The flow loop includes two vertical adiabatic pipes, one heat source at the bottom, and one heat sink on the top. Both the heat source and sink sections have the same pipe length, Δs. In the original derivations, several assumptions were made to simplify the problem: (1) the flow is incompressible; (2) the Boussinesq approximation is valid; (3) the wall friction force is proportional to flow rate; (4) the fluid temperature is uniform over any cross-section; (5) the heat flux between the pipe wall and the fluid is proportional to the temperature difference between them. Based on these assumptions, the Welander problem was formulated in a one-dimensional approach for both loop flow rate and temperature distribution in the loop. The momentum equation is formulated in an integral form,

$$\frac{dq}{dt} = \frac{A}{L} g\alpha \oint T dx - Rq \tag{1}$$

where, $q = uA$ is the flow rate; g is the gravity constant; α is the thermal expansion coefficient of the fluid; u is the fluid velocity; T is the fluid temperature; and R is the wall friction coefficient.

The energy equation is given for the heated and cooled sections only, in terms of fluid temperature,

$$\frac{\partial T}{\partial t} + \frac{q}{A}\frac{\partial T}{\partial x} = k(T_{wall} - T) \tag{2}$$

where, T_{wall} is the wall temperature, and $T_{wall} = \Delta T$ for the heat source section, $T_{wall} = -\Delta T$ for the heat sink section; k is a constant related to wall heat transfer. In the original paper (Welander, 1967), by taking advantage of the antisymmetric distribution, it was proved that, the steady motion and temperature distribution of the closed loop could be formulated as,

$$\epsilon \bar{q}_s = a \bar{T}_s \tag{3}$$

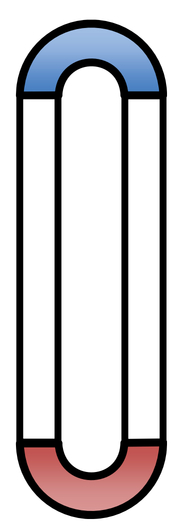

Fig. 1. Schematic drawing of the Welander problem.