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An Optimization Algorithm based on No-fit Polygon Method and Hybrid Heuristic Strategy for Irregular Nesting Problem

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Abstract: For the irregular nesting problem widely existing in modern manufacturing industry, this paper makes a research on it and presents an optimization algorithm based on no-fit polygon (NFP) method and hybrid heuristic strategy to solve it. The proposed algorithm first uses the composition method of trace line segment to calculate no-fit polygons (NFPs) between every two pieces in piece set, and extracts the candidate placement points for a candidate piece to be placed by using generated NFPs in combination with internal no-fit polygon (INFP) between the piece and the material plate. Then, the algorithm designs and applies three hybrid heuristic strategies to evaluate all candidate pieces and choose the best piece to place next and determine the best placement point for the selected piece. Experimental test has been performed to verify feasibility and effectiveness of the proposed algorithm. The test results show that the algorithm can solve the irregular nesting problem effectively, and improve the utilization of material to a certain extent.

Key Words: Irregular nesting, No-fit polygon, Hybrid heuristic strategy, Optimization algorithm

1 Introduction

The irregular nesting problem, also known as cutting problem or packing problem, widely exists in raw material processing industries, such as metal, clothing, wood, leather, stone and so on. Irregular nesting problem belongs to a kind of combinatorial optimization problems with geometric constraint. For the problem, one or more pieces of material are required to be divided into small pieces with specific shapes. It can also be described formally as that a list of given pieces with specific shapes are to be placed onto a given material plate, with no overlap between any two pieces and no piece beyond the scope of plate. The objective of the problem is to maximize the utilization of the plate.

Research on irregular nesting problem mainly focuses on two aspects: how to choose a proper placement position for the next piece to place onto the plate (corresponding to placement strategy), and how to choose a piece to be placed next (corresponding to sorting strategy).

The traditional placement strategies include the frontline method[1], the scanning-line method[2] and the no-fit polygon (NFP) method[3]. The frontline method needs to construct the outer contour of all placed pieces, then searches feasible placement positions for the next piece relative to the contour, and finally chooses an optimal placement position to place a piece. Its disadvantage is that the gap between placed pieces often cannot be used even for small pieces, which results in lower utilization of the material. Though the scanning-line method can identify any unused space (including hollow concaves and even holes) and make full use of them, it needs to discretize space of pieces and material by using a number of scanning lines. But, the main problem for this method is contradiction between accuracy and efficiency of computation. Too large scanning space would distort the graphics of pieces and material, causing low computation accuracy, and too small scanning space would increase computation complexity rapidly, causing low computation efficiency. The NFP method calculates the feasible touching points between two pieces using original graphical information of pieces as to keep accurate calculation. It can also handle special space such as holes and hollow concaves, leading to strong adaptability. Therefore, We use NFP method as the basic graphic tool for calculating feasible touching positions between pieces, and combines it with heuristic strategy to design the optimization algorithm for the whole nesting problem.

Because pieces are placed onto the plate one by one, sorting strategy is needed for determining placement order of all pieces. Even for a same placement strategy, different placement orders would lead to different nesting results. The piece sorting problem belongs to combinatorial optimization problem, which is known as non-deterministic polynomial complete problem. In the existing literatures, heuristic algorithms[4-5], local search algorithms[6] or global search algorithms[7] have been used to solve the problem. Among them, the heuristic algorithms optimize placement order of pieces by introducing heuristic rules or criteria, having advantages of simple implementation and low time complexity. However, heuristic algorithms with simple strategy have poor adaptability and may generate good or bad results for different nesting cases. More comprehensive heuristic strategy should be considered and designed to generate consistent good results for nesting problems.

This paper studies the NFP calculation method based on trace line segments, and proposes a hybrid heuristic strategy for searching an appropriate piece to place next and the best placement position for it in combination with NFP method, so as to solve the irregular nesting problem integrally.

2 Problem Formulation

The irregular nesting problem can be described formally as: Given a list of pieces $P = (p_1, p_2, \dots, p_n)$, and a rectangle plate of material $M = (W, L_{used})$ where W denotes the fixed width of the plate and L_{used} denote the length of plate been used after all pieces have been placed. The position of p_i is

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described by a vector $v_i = (x_i, y_i)$ of a predefined reference point on p_i. Shapes of pieces may be regular or irregular. For convenience of description, the length direction of the plate is set to be parallel to X-axis.

According to the requirements of nesting applications, nesting problem must satisfy the following constraints:

(1) No overlap between any two pieces.

(2) No piece exceeds the plate's boundary.

The irregular nesting problem is mathematically described as follows:

$$\begin{aligned} &maxU = Area_{all}/W * L_{used} \\ &s.t. p_i \cap p_j = \varphi, p_i \in C, i \neq j, i, j = 1,2,...n \end{aligned} \tag{1}$$

where $Area_{all}$ denotes the sum of areas of all pieces.

The constraints mentioned above indicate that there is no overlap between pieces and all pieces should be located inside the plate. The objective of optimization problem is to maximize the plate utilization, namely, to minimize the used length of the plate.

3 Generate Feasible Placement Points by No-fit Polygon Method

Before describing the hybrid heuristic algorithm, some basic geometrical concepts and calculation methods about no-fit polygon and the method of generating feasible placement points are first introduced in this section.

3.1 No-fit polygon

The concept of the no-fit polygon (NFP) was first introduced by Art[8]. It's used to judge whether there has overlap between two pieces. The no-fit polygon of polygon B relative to polygon A (named NFP_{AB}) is defined as one or several closed polygons resulted from movement of one predefined reference point on B while polygon B make a translational movement around the outer contour of polygon A, keeping polygon A stationary and polygon B touching A but without overlap during B's movement.

At present, the traditional methods for calculating no-fit polygon are collision method[9], Minkowski-sums method[10] and Ghosh's slope-diagram method[11]. Collision method, which originated from definition of NFP is simple and intuitive conceptually, can't handle cases involving polygons with hollow concave. Minkowski-sums method can only handle cases with only one concave polygon. Slope-diagram method can calculate NFP between two concave polygons, but has high time complexity of computation. For these reasons, here we apply a new method based on idea of trace line segment[12] to calculate no-fit polygon between any two polygons.

Our method for calculating NFP based on combination of trace line segments takes the following steps (for convenience, the polygons A and B are represented by sets of their counterclockwise vertexes):

Step 1: Specify a reference point on polygon B. The reference point could be any point on polygon B. For convenience of implementation, we often choose a vertex (especially vertex with an extreme coordinate) on B as the reference point.

Step 2: Judge touching state between angles and edges of two polygons. First judge whether it is touchable or not between every angle of one polygon and every edge of another polygon, and then construct all touchable "angle-edge" pairs.

The judgement rule can be described as follows: translate the edge to make the starting vertex of the edge be coincided with the vertex of the angle, and judge whether the edge is in the scope of angle area between the entering-edge of the angle and the reverse extension line of the departing-edge of the angle. If so, they are touchable; otherwise, they are not. Fig. 1 shows an example of touching judgement between an angle and an edge. According to the rule, the angle $\angle AOB$ can touch the edge $\overrightarrow{EF}$.

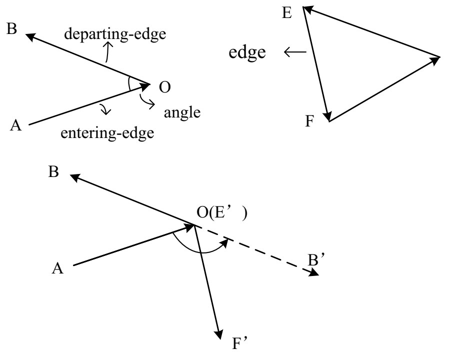
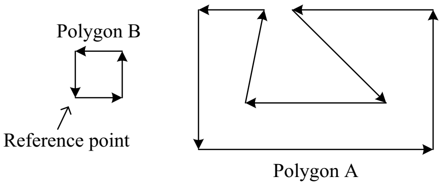


Fig. 1: Touching judgement between an angle and an edge

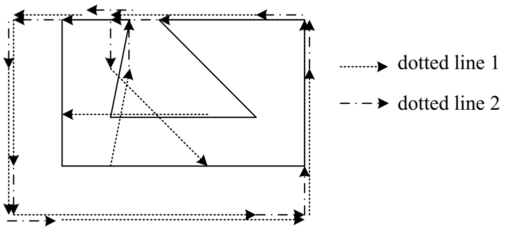
Step 3: Generate the set of trace line segments. Using all touchable "angle-edge" pairs, a set of trace line segments can be generated by the following method:

(1) If the angle belongs to polygon A and the edge belongs to polygon B, translate B to make the starting vertex of the edge be coincided with the vertex of the angle, and move B along the opposite direction of the edge until the ending vertex of the edge reaches the angular vertex. The locus generated by movement of the reference point on B is a so-called trace line segment.

(2) If the angle belongs to polygon B and the edge belongs to polygon A, translate B to make the angular vertex be coincided with the starting point of the edge, and move B along the edge until the angular vertex reaches the ending vertex of the edge. The locus generated by movement of the reference point is also a valid trace line segment.



(a) Two polygons: A and B



(b) Trace line segments formed by B relative to polygon A

Fig. 2: Trace line segments between two polygons