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Operator-based linearization for general purpose reservoir simulation

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ABSTRACT

General purpose reservoir simulation is based on the solution of governing equations describing mass and energy transfer in the subsurface. The solution process requires the linearization of strongly nonlinear governing equations. Usually, a Newton-based method is used for the linearization. This method demands the assembly of Jacobian and residual for a fully coupled system of equations. Recently, a new linearization approach was proposed and tested for binary systems. The key idea of the Operator Based Linearization (OBL) approach is to transform the discretized mass and energy conservation equations to an operator form which separates space-dependent and state-dependent properties of governing equations. This transformation provides the opportunity to approximate the representation of exact physics (physical properties) of a problem. Specifically, each term of the conservation equations is presented as the product of two different operators. The first operator depends on the current physical state of a system and contains fluid properties, such as density, viscosity, relative permeability, etc. The second operator captures both spatially altered properties, such as permeability, and the rest of state variables, such as pressure in the discrete approximation of gradient. All state-dependent operators are uniformly parametrized within the physical space of the problem (pressure-composition intervals). During simulation process, a multilinear interpolation is applied to approximate the first type of operators, while the second type of operators is processed based on conventional approach. In this work, we extended the approach to thermal systems with an arbitrary number of components. Besides, we significantly improved the performance of OBL employing adaptive parametrization technique. We tested the approach for truly multi-component thermal systems of practical interest. The computational performance, accuracy, and robustness of a new method were demonstrated against the conventional approach.

1. Introduction

Numerical simulations are essential for the modern development of subsurface reservoirs (Aziz and Settari, 1979). They are widely used for the evaluation of oil recovery efficiency, performance analysis, and various optimization problems. Due to the complexity of underlying physical processes and considerable uncertainties in the geological structure of reservoirs, there is a persistent demand for accurate and efficient models. In order to increase the accuracy of a model, one can apply a finer computational grid in space or time, or use a more detailed description of the fluids such as in thermal-compositional model. However, the improvement in the accuracy of models is usually counterbalanced by the reduction in the turnaround time of simulation. In the presence of ensemble optimization or stochastic solution based on a version of Monte Carlo approach, demanding thousands of simulations, the performance of forward simulation becomes a primary issue (Muller et al., 2016).

Space and time approximations usually introduce nonlinearity to the system of governing equations, further enhanced by complex behavior of multiphase fluid flow. Numerical solution of such systems with millions of unknowns is the only known way to complete simulation in feasible time. The particular set of independent variables (i.e., nonlinear unknowns) is defined by the nonlinear formulation of the actual simulation framework (Cao, 2002). During linearization stage, all properties and their derivatives need to be determined with respect to nonlinear unknowns. The linearization of the nonlinear system requires Jacobian assembly and consumes a significant portion of simulation time, especially for complex physical processes (Zaydullin et al., 2016).

Several conventional linearization approaches exist, though neither of them is robust, flexible, and computationally efficient all at once. Numerical derivatives provide flexibility in the nonlinear formulation (see Xu et al., 2011 for example), but a simulation based on numerical derivatives may lack robustness and performance (Vanden and

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Orkwis, 1996). Straightforward hand-differentiation is the workhorse strategy in modern commercial simulators (Schlumberger, 2007; Cao et al., 2009). However, this approach requires introduction of a complicated framework for storing and evaluating derivatives for each physical property, which in turn reduces the flexibility of a simulator to incorporate new physical models and increases probability for potential errors. The development of Automatic Differentiation (AD) technique allows preserving both flexibility and robustness in derivative computations. In reservoir simulation, the AD-based library (ADETL) was introduced by Younis (2011). Using the capabilities of ADETL, the Automatic Differentiation General Purpose Research Simulator (ADGPRS) was developed (Voskov and Tchelepi, 2012; Zhou et al., 2011). Later, the AD technique becomes more demanded in research frameworks for reservoir simulation (Krogstad et al., 2015). Being attractive from the perspective of flexibility, the AD technique by design inherits computational overhead, which affects the performance of reservoir simulation (Khait and Voskov, 2017).

A novel linearization approach called Operator-Based Linearization (OBL), where performance, robustness, and flexibility can be combined without compromise, was introduced in Voskov (2017). Each term in discretized conservation equations is represented by the product of two operators: state- and space-dependent. The state-dependent operators are adaptively parametrized over the physical space of a simulation problem, while space-dependent operators are applied in the conventional manner. During the course of the simulation, the state-dependent operators are calculated based on the multilinear interpolation in multidimensional space of nonlinear parameters. The performance gain of Jacobian assembly with OBL reaches an order of magnitude (Khait and Voskov, 2017).

In this paper, we extend the OBL method to a general purpose thermal-compositional reservoir simulation. ADGPRS is used as an implementation framework and as the reference approach for fidelity and performance comparisons. We apply the extended OBL to the several reservoir simulation problems of practical interest. Better nonlinear performance with the coarser representation of physics is demonstrated, while the approximation error is controlled by the resolution of the interpolation tables. Several advantages and extensions of the proposed method are discussed in the conclusion.

2. Conventional modeling approach

In this section, we describe one of the conventional nonlinear formulations for a general purpose thermal-compositional model. This formulation was implemented in ADGPRS (Voskov and Tchelepi, 2012) and is used in this paper as the reference solution.

2.1. Governing equations

Here, we describe the flow of energy and mass in a system with n_p phases and n_c components. For this model, n_c component mass conservation equations and a single energy conservation equation need to be written as

$$\frac{\partial}{\partial t}\left(\phi \sum_{p=1}^{n_p} x_{cp}\rho_p s_p\right) + \mathrm{div}\sum_{p=1}^{n_p} x_{cp}\rho_p \vec{u}_p + \sum_{p=1}^{n_p} x_{cp}\rho_p \tilde{q}_p = 0, \quad c = 1, \ldots, n_c, \tag{1}$$

$$\frac{\partial}{\partial t}\left(\phi \sum_{p=1}^{n_p} \rho_p s_p U_p + (1-\phi)U_r\right) + \mathrm{div}\sum_{p=1}^{n_p} h_p\rho_p \vec{u}_p + \mathrm{div}(\kappa\nabla T) + \sum_{p=1}^{n_p} h_p\rho_p \tilde{q}_p = 0. \tag{2}$$

All terms of the system (1)–(2) can be characterized as functions of the spatial coordinates $\boldsymbol{\xi}$ and physical state $\boldsymbol{\omega}$ as follows:

- $\phi(\boldsymbol{\xi}, \boldsymbol{\omega})$ – effective rock porosity,
- $x_{cp}(\boldsymbol{\omega})$ – component concentration in phase,
- $\rho_p(\boldsymbol{\omega})$ – phase molar density,
- $s_p(\boldsymbol{\omega})$ – phase saturation,
- $\vec{u}_p(\boldsymbol{\xi}, \boldsymbol{\omega})$ – phase velocity,
- $\tilde{q}_p(\boldsymbol{\xi}, \boldsymbol{\omega}, \mathbf{u})$ – source of phase,
- $\mathbf{U}_p(\boldsymbol{\omega})$ – phase internal energy,
- $\mathbf{U}_r(\boldsymbol{\xi}, \boldsymbol{\omega})$ – rock internal energy,
- $h_p(\boldsymbol{\omega})$ – phase enthalpy,
- $\kappa(\boldsymbol{\xi}, \boldsymbol{\omega})$ – thermal conduction.

The only exception here is the phase source term which also depends on $\mathbf{u}$ – well control variables.

Next, for simplicity, we assume that the rock internal energy and thermal conduction are spatially homogeneous, thus

$$U_r = f(\boldsymbol{\omega}), \quad \kappa = f(\boldsymbol{\omega}). \tag{3}$$

Phase flow velocity is assumed to follow the Darcy law as

$$\vec{u}_p = -\left(\mathbf{K}\frac{k_{rp}}{\mu_p}\left(\nabla p_p - \vec{\gamma}_p \nabla D\right)\right), \tag{4}$$

where

- $\mathbf{K}(\boldsymbol{\xi})$ – effective permeability tensor,
- $k_{rp}(\boldsymbol{\omega})$ – phase relative permeability,
- $\mu_p(\boldsymbol{\omega})$ – phase viscosity,
- $p_p(\boldsymbol{\omega})$ – phase pressure,
- $\vec{\gamma}_p(\boldsymbol{\omega})$ – gravity vector,
- $D(\boldsymbol{\xi})$ – depth (backward oriented).

After application of a finite-volume discretization on a general unstructured mesh and backward Euler approximation in time, we get

$$V\left(\left(\phi\sum_{p=1}^{n_p} x_{cp}\rho_p s_p\right)^{n+1} - \left(\phi\sum_{p=1}^{n_p} x_{cp}\rho_p s_p\right)^{n}\right) - \Delta t\sum_{l}\left(\sum_{p=1}^{n_p} x_{cp}^{l}\rho_p^{l}\Gamma_p^{l}\Delta\psi^{l} + \Delta t\sum_{p=1}^{n_p} x_{cp}\rho_p \mathbf{q}_p = 0\right), \tag{5}$$

$$V\left[\left(\phi\sum_{p=1}^{n_p}\rho_p s_p U_p + (1-\phi)U_r\right)^{n+1} - \left(\phi\sum_{p=1}^{n_p}\rho_p s_p U_p + (1-\phi)U_r\right)^{n}\right] - \Delta t\sum_{l}\left(\sum_{p=1}^{n_p} h_p^{l}\rho_p^{l}\Gamma_p^{l}\Delta\psi^{l} + \Gamma_c^{l}\Delta T^{l}\right) + \Delta t\sum_{p=1}^{n_p} h_p\rho_p\mathbf{q}_p = 0, \tag{6}$$

where V is the control volume of a grid cell and $q_p = \tilde{q}_p V$ is the source of a phase p. Here, we have neglected capillarity, gravity and used Two-Point Flux Approximation (TPFA) with upstream weighting. Therefore, $\Delta\psi^{l}$