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Effects of initial curvature on coiled tubing buckling behavior and axial load transfer in a horizontal well

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ABSTRACT

This paper builds an analytical model to describe the comprehensive buckling behavior of coiled tubing (CT) with initial curvature in a horizontal well. The new buckling equation and contact force considering the effect of initial curvature is built on the basis of beam-column theory. With the new contact force expression, the work done by lateral friction can be obtained, thus the total potential energy of CT with initial curvature can be calculated. By use of the principle of virtual work and energy conservation, the critical sinusoidal and helical buckling load can be obtained. Moreover, the effect of initial curvature on axial load transfer is also calculated by a new axial load equation. The calculation results indicate that initial curvature has a strong influence on buckling loads and a CT with initial curvature is less efficient for axial load transfer compared to a straight CT. To verify the proposed model, the results of this paper are compared to experimental results, which support the proposed solutions.

1. Introduction

The buckling behavior of tubular has been studied for more than half centuries. Lubinski (1950); Lubinski and Althouse, 1962) first systematically analyzed the 2D lateral buckling and 3D helical buckling of drill string in vertical wells, and derived the relationship between the critical axial load and the pitch of helix. Dawson and Paslay (1984) derived the first, now well-known, expression [Eq. (1)] for the critical sinusoidal buckling load of a tubular constrained in an inclined wellbore.

$$F_{crs} = 2\sqrt{\frac{EIq \sin\phi}{r_c}} \tag{1}$$

where F_{crs} is the critical sinusoidal buckling force, q is the tubular weight per unite length, ϕ is the inclination angle of the wellbore, EI is the bending stiffness, r_c is the radial clearance between the tubular and wellbore.

Mitchell (1988) first established the buckling equation describing a pipe constrained in an inclined wellbore. He also derived the tubular contact force.

$$EIr_c\left[\frac{d^4\theta}{dz^4} - 6\left(\frac{d\theta}{dz}\right)^2\frac{d^2\theta}{dz^2}\right] + r_c\frac{d}{dz}\left(F\frac{d\theta}{dz}\right) + q\sin\phi\sin\theta = 0$$

$$N = EIr_c\left[4\frac{d\theta}{dz}\frac{d^3\theta}{dz^3} + 3\left(\frac{d^2\theta}{dz^2}\right)^2 - \left(\frac{d\theta}{dz}\right)^4\right] + Fr_c\left(\frac{d\theta}{dz}\right)^2 + q\sin\phi\cos\theta$$

where θ is the angular displacement, N is the normal contact force. F is the axial force and z is the coordinate along the wellbore axis.

From then on many researchers found the same formula [Eq. (1)] for critical sinusoidal buckling force. However, unlike the consistent critical buckling point between the initial straight configuration and sinusoidal configuration, there is no consistent model to represent the transition from sinusoidal to helical buckling (Miska et al., 1996; Mitchell, 1997). Chen et al. (1990) first derived the expression of the critical helical buckling load in a horizontal well. Wu (1992) pointed out that Chen's result was an average value of the critical sinusoidal and helical buckling loads, and deduced another buckling load. In the next few years, Gao et al. (1998), Gao and Huang (2015), Liu (1999) and Huang et al. (2015a, 2015b) studied buckling behaviors by using both the energy method and the tubular-buckling equations. Table 1 summarizes the values of the critical buckling loads proposed by the above researchers. These different forms of critical buckling loads are deduced under different assumptions about the axial load in the entire loading process.

At the same time, effects of friction on tubular critical buckling loads have been studied analytically and experimentally. Mitchell (1988, 2007) studied the complexity of friction and derived the critical loads for two simple cases with friction in vertical wells. Suryanarayana

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Nomenclature

A_0	initial amplitude of the sinusoidal configuration, radians
A	final amplitude of the sinusoidal configuration, radians
EI	bending stiffness, lbf-in2
f_1	axial friction coefficient
f_2	lateral friction coefficient
F	axial compression force, lbf
q	tubular weight per unite length, lbf/ft
L	length of CT, ft
ϕ	inclination angle of the wellbore, degrees
r_c	radial clearance between CT and wellbore, ft
θ_0	CT initial angular displacement, radian
θ	CT final angular displacement, radian
p_0	initial "wave-length" of CT, 1/ft
p_s	final "wave-length" of CT, 1/ft
p_h	final "pitch" of CT, 1/ft
N	normal contact force, lbf
x_0, y_0	initial lateral displacements, ft
x, y	final lateral displacements, ft
C_1, C_2	functions of initial CT configuration
C_3, C_4	dimensionless functions of initial CT configuration
$\boldsymbol{h}$	distributed force, lbf
$\boldsymbol{M}$	moments on the cross section of the CT, lbf-ft
β	dimensionless axial load
β_L	dimensionless axial load at the loading end
β_D	dimensionless axial load at the dead end
U	elastic deformation energy, lbf-ft
W	work done by external force, lbf-ft
$\prod$	total potential energy, lbf-ft
Ω	dimensionless total potential energy
k_1	number of half-sinusoidal waves
k_2	number of complete helix turns

Subscripts

crs	critical value for sinusoidal buckling
crh	critical value for helical buckling
f	friction
b	bending

and McCann (1995) and McCann and Suryanarayana (1994) conducted experiments on pipe buckling with friction. Gao and Miska (2009, 2010) investigated the effects of friction on pipe buckling in a horizontal well, and the results showed that friction had increased the buckling load by 30–50%. Recently, Su et al. (2013) presented a theory to reveal the initiation of rod instability under initial velocity.

However, all these models typically assumed that the tubular was initially straight in the wellbore. This assumption is suitable for tubulars like dill pipe, tubing, casing and so on, but for coiled tubing (CT). In fact, all real CT has minor initial bending. During operations, the CT is unspooled from the reel and bent on the gooseneck. Throughout the process, CT goes through four times bending-straighten deformation, and every bending deformation makes the CT into plastic state resulting in residual bending. After entering into the wellbore, the CT is not straight but has an initial configuration.

As to our knowledge, only several studies considered the effect of residual bending. Miska et al. (1996) observed the effect of residual bending on pipe in experiment. Qiu et al. (1997, 1999) established a model to analyze the effect of CT initial configuration on sinusoidal and helical buckling behavior in deviated wells. Zheng and Adnan (2005) also noticed the questions of residual bending in CT, and he assumed the initial configuration of CT was in the form of a helix. However, the friction was neglected in all of these models.

The common practice to handle CT residual bending is to increase the friction coefficient in conventional model to account for it. But how does residual bending exactly affect the critical buckling loads under the condition of friction? How does residual bending affect the axial load transfer? It is important for us to answer these two questions. In this paper, we first focus on the buckling analysis about the critical conditions (or critical loads) above which CT will change its configuration from one type into another. By using beam-column method, new governing differential equation and normal contact force are derived. In order to get the analytical solutions, the critical sinusoidal and helical buckling loads are derived with the energy method. Since the contact force between CT and wellbore is obtained, the effect of friction can be considered as a dissipative term, and incorporated into the generalized potential. When the initial CT configuration is straight, buckling solutions of the new equations are identical with previous conventional results. Secondly, by considering the axial friction, the effects of initial curvature on axial load transfer are also analyzed. The model proposed in this paper does not intend to replace, but rather complements existing domain models. Through these analysis, we find the initial curvature has a significant effect on CT buckling behavior, and these new results allow for accurate job design to operate CT in the wellbore.

Table 1
the values (F/F_{crs}) of critical buckling loads for different buckling models.

Researchers	Straight	Sinusoid	Transition	Helix
Chen et al. (1990)	[0, 1]	$[1, \sqrt{2}]$	/	$[\sqrt{2}, \infty]$
Wu (1992)	[0, 1]	$[1, 2\sqrt{2}-1]$	/	$[2\sqrt{2}-1, \infty]$
Miska et al. (1996)	[0, 1]	[1, 1.875]	$[1.875, 2\sqrt{2}]$	$[2\sqrt{2}, \infty]$
Mitchell (1997)	[0, 1]	$[1, \sqrt{2}]$	$[\sqrt{2}, 2\sqrt{2}]$	$[2\sqrt{2}, \infty]$
Gao et al. (1998)	[0, 1]	[1, 1.401]	/	$[1.401, \infty]$

2. Mathematical model

2.1. Major assumptions

In order to build the tubular analysis model, the following major assumptions are applied.

1. The wellbore is a horizontal straight cylinder.
2. The CT is in continuous contact with the wellbore wall. In the sinusoidal buckling stage, the CT snakes along the low surface of the wellbore. While in the helical buckling stage, the CT buckles as a helix spiraling around the inner surface of the wellbore wall.
3. The slender-beam theory is used to relate bending moment to curvature.
4. The initial amplitude of CT angular displacement is small.
5. At the onset of instability only one buckling mode dominates.

2.2. Geometric description

The O-xyz coordinates are shown in Fig. 1. The origin of the Cartesian coordinates is set at the center of the cross section of the wellbore at the leftmost end. The z axis points horizontally from left to right along the axis of the wellbore. The x axis points vertically

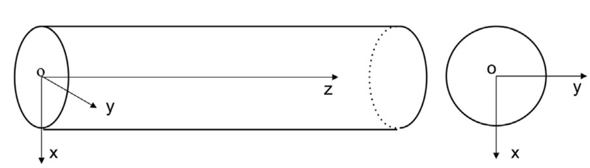

Fig. 1. Coordinate system for buckling analysis.