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A model ranking and uncertainty propagation approach for improving confidence in solids transport model predictions

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ABSTRACT

The transport of solid particles in pipelines is of interest in the petroleum industry, and is needed to increase flow efficiency in the pipe and prevent pipeline damage due to the particles' accumulation. To achieve this goal, the velocity of the carrier fluid in the pipe needs to exceed the threshold velocity. Many solids transport models are available for predicting the threshold velocity, but for the same input condition, the predictions of these models may vary by orders of magnitude, and information regarding the confidence of the models' predictions is not readily available. To resolve these issues, this paper presents a model evaluation and uncertainty propagation approach that uses a novel combination of data clustering, model parameter fine-tuning, model screening and ranking, model uncertainty quantification, and Monte Carlo simulation methods. The inputs are the experimental database for solids transport, a set of solids transport models, and the input condition(s) where the models' predictions are needed. The outputs of the methodology include the models' rankings, and the envelopes of the models' predictions to within a predetermined confidence level. By propagating the uncertainties of the models, experimental data, and input conditions, the highest-ranked models produce velocity envelopes at the 90% confidence level that cover the experimentally-observed values for 92% of the cases; while using the prediction of an individual model does not provide any information regarding the prediction confidence.

1. Introduction

In the petroleum industry, the need to hydraulically transport solid particles is encountered frequently. For instance, hydraulic fracturing involves injecting fluid [typically water, oil, acids, methanol (Pangilinan et al., 2016), or water mixed with drag-reducing polymer (Gu and Mohanty, 2015)] and proppants [typically sand, ceramic, or resin-coated ceramic or sand (Pangilinan et al., 2016)] at high rate and pressure (Shiozawa and McClure, 2016). This process creates fractures in the "geologic formations" (Pangilinan et al., 2016), which increases the permeability and the production rate of the oil reservoir (Zheng et al., 2015). In another application, during well drilling, the cuttings need to be transported by the drilling fluid (Akhshik et al., 2015) to prevent the formation of a stationary bed of solids at the bottom of the wellbore (Rodriguez Corredor et al., 2016). Consequences of having a stationary bed of solids include "slow drilling rate, and in severe cases, stuck pipe" (Rodriguez Corredor et al., 2016).

In these cases, the fluid velocity must exceed the threshold velocity to successfully transport the solid particles in the pipe. Many solids transport models exist that predict such velocity (Soepyan, 2015). Furthermore, different threshold velocity definitions exist (Soepyan et al., 2014), including the critical velocity (the fluid velocity that marks the boundary between the settling of solid particles at the bottom of the pipe and the particles' full suspension) (Oroskar and Turian, 1980), saltation velocity (the minimum fluid velocity needed to prevent suspended solid particles from settling to the bottom of the pipe) (Zenz, 1964), equilibrium velocity (the fluid velocity where the rate at which the particles are transported by the fluid equals the rate at which the particles settle to the bottom of the pipe) (Gruesbeck et al., 1979), pick-up velocity (the fluid velocity required to initiate the motion of a solid particle initially at rest on a bed of solids) (Hayden et al., 2003), and incipient motion velocity (the fluid velocity required to initiate the motion of a solid particle initially at rest at the bottom of the pipe) (Rabinovich and Kalman, 2009a).

Different models may be developed using different assumptions regarding the dominant forces for solid particle transport, given the

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Nomenclature

C	particle volumetric concentration
C_i	particle volumetric concentration of datum point i
C_0	particle volumetric concentration of the input condition
d_i	weighted Euclidean distance between experimental datum point i and the input condition
D	hydraulic diameter of the conduit
D_i	hydraulic diameter of the conduit of datum point i
D_0	hydraulic diameter of the conduit of the input condition
d_p	particle diameter
$d_{p,i}$	particle diameter of datum point i
$d_{p,0}$	particle diameter of the input condition
$E_{MA,j}$	mean absolute error of model j
$E_{MAP,j}$	mean absolute error percentage of model j
$E_{MS,j}$	mean squared error of model j
$E_{P,i,j}$	error percentage of model j at experimental datum point i in the reduced database
$E_{SS,j}$	error sum of squares of the threshold velocity predictions of model j
$F_{calc}(v)$	distribution function of the predicted threshold velocity in the reduced database
$F_{exp}(v)$	distribution function of the experimentally-observed threshold velocity in the reduced database
$f(\underline{x_l}, \underline{k})$	equation of the model
$f\left(\underline{x_{l,i}}, \underline{k_j}\right)$	equation of model j at datum point i
H_a	alternative hypothesis
h_l	correlation between independent variable l and the threshold velocity
H_0	null hypothesis
i	index of the experimental data points
j	index of the models
J	number of ranked models
$\underline{k}$	vector that consists of the parameters (constants) of the model
k_j	number of parameters in model j
$\underline{k_j}$	vector that consists of the parameters of model j
$k_{j,non}$	number of parameters in model j that become non-zero after the model parameter fine-tuning process
l	index of the independent variables
$max_{\%,j}$	maximum between $over_{\%,j}$ and $under_{\%,j}$
m_j	slope between the predictions of model j and the experimentally-observed values of the threshold velocity
$m_{0,j}$	slope between the predictions of model j and the experimentally-observed values of the threshold velocity, with the intercept forced to be at the origin
N_{Ar}	Archimedes number
$N_{Ar,i}$	Archimedes number of datum point i
$N_{Ar,0}$	Archimedes number of the input condition
n_{data}	number of data points in the reduced database
N_{data}	number of data points in the experimental database
$n_{indep,j}$	number of independent variables incorporated in model j
N_{model}	total number of models available in the model database
$N_{Re,p}$	particle Reynolds number
$N_{Re,p,i}$	particle Reynolds number of datum point i
$N_{Re,p,i,j}$	particle Reynolds number predicted by model j for datum point i
n_{trial}	total number of trials (replications) for the Monte Carlo simulation method
N_{var}	total number of independent variables that describe the physical system
$over_{\%,j}$	percentage of experimentally-observed threshold velocity overestimated by model j in the reduced database
$P_{\%,j}$	percentage of threshold velocity predictions produced by model j that lie outside $\pm(100\%) \times \min(1 - \varepsilon_l,\ \varepsilon_u - 1)$ of the experimental observations
R_j^2	R^2 statistic of model j
$R^2_{adj,dev,j}$	deviation of the modified adjusted-R^2 statistic of model j from the value of one
$R^2_{adj,j}$	adjusted-R^2 statistic of model j
$R^2_{adj,mod,j}$	modified adjusted-R^2 statistic of model j
S_j	score of model j
t	index of the trial (replication) of the Monte Carlo simulation method
T_{SS}	total sum of squares of the experimentally-observed threshold velocity
$T_{SS,mod}$	modified total sum of squares of the experimentally-observed threshold velocity
T_1	test statistic for the null hypothesis
$U_{exp,i}$	uncertainty of the experimentally-observed threshold velocity at datum point i
$under_{\%,j}$	percentage of experimentally-observed threshold velocity underestimated by model j in the reduced database
U_{x_l}	uncertainty of independent variable l
$U_{x_{l,i}}$	uncertainty of independent variable l at datum point i
v	threshold velocity
$v_{calc,i,j}$	threshold velocity predicted by model j for experimental datum point i
v_{exp}	experimentally-observed threshold velocity
$v_{exp,avg}$	average value of the experimentally-observed threshold velocity in the reduced database
$v_{exp,i}$	experimentally-observed threshold velocity of datum point i
$v_{L,exp,i}$	lower bound of the value of the threshold velocity at experimental datum point i
$v_{M,i,j}$	estimated “true” value of the threshold velocity at the input condition given the error of model j at experimental datum point i
$v_{U,exp,i}$	upper bound of the value of the threshold velocity at experimental datum point i
$v_{0,avg}$	average value of the threshold velocity predictions of all the ranked models for the input condition
$v_{0,dev,j}$	absolute deviation of $v_{0,j}$ from $v_{0,avg}$
$v_{0,i}$	threshold velocity prediction of the model for the ith input condition
$v_{0,j}$	threshold velocity prediction of model j for the input condition
x	independent variable
x_l	value of independent variable l
$\underline{x_l}$	vector that contains the values of the independent variables
$x_{l,i}$	value of independent variable l at datum point i
$\underline{x_{l,i}}$	vector that contains the independent variables of datum point i
$\overline{x_{l,i}}$	normalized $x_{l,i}$
$x_{L,l,i}$	lower bound of the value of independent variable l at experimental datum point i
$x_{l,0}$	value of independent variable l at the input condition
$\overline{x_{l,0}}$	normalized $x_{l,0}$
$x_{U,l,i}$	upper bound of the value of independent variable l at experimental datum point i
x_1	first independent variable
x_2	second independent variable
y_j	statistic of model j
z	dependent variable
α_S	level of significance
ε_l	acceptable lower bound of the ratio of the model's prediction to the value of the threshold velocity observed experimentally
ε_u	acceptable upper bound of the ratio of the model's predic-