Journal of Petroleum Science and Engineering 151 (2017) 496–506

Contents lists available at ScienceDirect

Journal of Petroleum Science and Engineering

journal homepage: www.elsevier.com/locate/petrol

# Inversion of fractures based on equivalent continuous medium model of fractured reservoirs

Kai Zhang*, Xiaoming Zhang, Liming Zhang, Jun Yao*, Xia Yan

*China University of Petroleum, 66 Changjiang West Road, Qingdao, Shandong 266555, China*

ARTICLE INFO

*Keywords:*
Inversion of fractures
Production performance
Bayesian theory
Equivalent continuous medium model
SPSA algorithm

ABSTRACT

Because the fractures in fractured reservoirs have a significant effect on the flow behavior of fluid, the production performance of the fractured reservoirs contains important information of the distribution of fractures. That is the basic principle of applying the production information for the inversion of fractures. It is the first step to link the distribution information of fractures with the production information to establish inversion objective function on the basis of Bayesian theory. And the geometric parameters of fractures such as midpoint coordinate $(x_0, y_0)$, azimuth $\alpha$ and extension length $L$ which directly determine the distribution of fractures and then affect the flow behavior are chosen as inversion parameters and some production data are chosen as inversion indexes. Secondly, in order to reconcile the demand of reflecting the flow effect of individual fractures and the use of mature simulators, equivalent continuous medium model whose equivalent permeability tensor is a crucial parameter reflecting the flow behavior of fractured porous media is applied for calculating the numerical simulation of the fractured reservoirs. Finally, SPSA algorithm is adopted to minimize the inversion objective function and then update the distribution of fractures to obtain the optimal results. Theoretical cases verify that the inversion method is effective to identify fractures while the number of identified fractures is limited because of the multi-solution of the inverse problem.

## 1. Introduction

The flow behavior in the fractured reservoirs is complicated and injected water or edge water can easily break through production well, which results in the poor development. Therefore, accurate identification of fractures is a crucial step to effectively develop the fractured reservoirs. Conventional techniques of identifying fractures such as core analysis, outcrop data, logging and seismic exploration are limited to identify the fractures near wellbores because of ignoring the effect of fractures on the flow behavior. Considering that the production performance of oilfields is useful to reflect the flow behavior, the production information is used to invert the distribution of fractures (Gang and Kelkar, 2006; Suzuki et al., 2005). The inversion objective function is established on the basis of Bayesian theory, which represents the difference between simulated production and actual production of oilfields. And an efficient optimization algorithm is used to minimize the difference and then optimal results which are consistent with the actual distribution of fractures are obtained (Zhao et al., 2013).

Firstly, the geometric parameters of fractures such as midpoint coordinate $(x_0, y_0)$, azimuth $\alpha$ and extension length $L$ of fractures are chosen as inversion parameters to directly connect the geometry of fractures with dynamic flow behavior. Secondly, it is necessary to adopt a proper mathematical flow model to simulate the flow behavior in the fractures reservoirs. There are three main flow models are used to describe the flow behavior in the fractures reservoirs: dual medium model, equivalent continuous medium model and discrete fracture model. Dual medium model was proposed by Barenblatt et al. It is much simplified and is not suitable to describe the flow effect of individual fractures (Zheltov et al., 1960). Discrete fracture model was proposed by Noorishad et al. and it has the advantage of explicitly describing the effect of fractures on the flow behavior of fluid. However, the model involves complicated fracture networks and it is difficult to be combined with mature simulators (Noorishad, 1982; Baca et al., 1984; Doe et al., 2013; Zhu et al., 2016). Equivalent continuous medium model was presented by Snow, which considers the fractured porous media as the porous continuous media with the help of the equivalent permeability tensor, which is used to reflect the flow effect in the equivalent porous continuous media (Snow, 1969). One of the advantages of the model is that it can both effectively reflect the flow effect of individual fractures and be combined with mature simulators based on continuous medium (Durlofsky, 1991; Teimoori et al., 2003). When it comes to minimize the inversion objective function, the

* Corresponding authors.
*E-mail address:* reservoirs@163.com (J. Yao).

http://dx.doi.org/10.1016/j.petrol.2017.01.015
Received 16 June 2016; Received in revised form 20 December 2016; Accepted 4 January 2017
Available online 06 January 2017

optimization algorithms that are applied for the inversion generally involve gradient and non-gradient methods. The gradient methods include Gauss-Newton method, Adjoint method and so on (Kalogerakis and Tomas, 1995; Wu et al., 1999; Rodrigues et al., 2006; Shirangi, 2013). The feature that the methods need to be fully combined with implicit reservoir simulator to calculate high-dimensional Jacobian Matrix limits the application of gradient methods for large-scale inversion of reservoirs parameters. In order to avoid calculating gradients, some non-gradient methods have received extensive attention, such as genetic algorithm, Evolutionary Algorithm, Particle Swarm Optimization, EnKF and SPSA, etc. (Sen et al., 1995; Abdelkhalik et al., 2012; Mohamed et al., 2011; Gu and Oliver, 2006; Gao et al., 2004; Zhao et al., 2016; Chen et al., 2016). Although the non-gradient methods can avoid calculating gradients, they have other disadvantages: genetic algorithm relies on thousands of simulation operations to obtain the optimal results; EnKF is not suitable for the inversion of highly nonlinear feature of the flow behavior in reservoirs. In order to make the inversion process easily achieved and guarantee the high efficiency of operation, SPSA optimization algorithm is adopted. SPSA (Simultaneous Perturbation Stochastic Approximation) was first introduced to achieve the inversion of petro-physical parameters by Gao et al. SPSA gradients are easy to obtain and are invariably descent direction in terms of minimization problems (Kamouei and Yildiz, 2007). In general, the mean of several SPSA gradients is used as search direction, and considering the independence of SPSA gradients, several SPSA gradients can be calculated on the same time based on parallel computation to enhance the computation efficiency (Zhang et al., 2015).

And the rest of the paper is organized as follows: Section 2 presents the principal idea of applying production performance for the inversion of fractures with subsection 2.1 establishing the inversion objective function based on Bayesian theory and subsection 2.2 preparing the inversion parameters and indexes. Section 3 introduces the equivalent continuous medium model to get simulated production. Section 4 adopts the SPSA optimization algorithm for updating the inversion parameters. Section 5 generalizes the whole inversion process and Section 6 illustrates the application of the inversion method for two theoretical cases. Finally, in Section 7 some conclusions are drawn.

## 2. Inversion of the distribution fractures

Relying on production performance for the inversion of fractures is an inverse problem: the distribution of fractures is updated to reduce the difference of simulated production with actual production of oilfields based on optimization algorithms and the optimal distribution of fractures is obtained in the end. Therefore, the establishment of a proper objective function, selection of useful inversion parameters and adoption of an effective optimization algorithm are all crucial for the inversion of fractures.

### 2.1. Inversion objective function

Assuming that reservoir parameters are random variables conforming to the multivariate Gaussian distribution, and there is a corresponding relationship between reservoir parameters and production data of oilfields (Tarantola, 2005). Therefore, on the basis of Bayesian theory, when the production $d_{obs}$ is given, the conditional probability distribution function of the reservoir parameters $m$ is given by:

$$f(m|d_{\text{obs}}) \propto f(d_{\text{obs}}|m) f(m) \tag{1}$$

The probability distribution function of the reservoir parameters $m$ is defined as:

$$f(m) \propto \exp\left[-\frac{1}{2}(m-m_{pr})^T C_M^{-1}(m-m_{pr})\right] \tag{2}$$

Where $m$ is a vector of $N_m$ dimension with mean value as $m_{pr}$ and covariance matrix as $C_M$, and $m_{pr}$ is the prior estimate of the reservoir parameters.

When the reservoir parameters $m$ is given, the conditional probability distribution function of the production is given by:

$$f(d_{\text{obs}}|m) \propto \exp\left[-\frac{1}{2}(d_{\text{obs}} - g(m))^T C_D^{-1}(d_{\text{obs}} - g(m))\right] \tag{3}$$

Where $d_{obs}$ is a vector of $N_d$ dimension, containing the production information such as oil production rate and water production rate and $C_D$ is the covariance of the production data. $g(\cdot)$ representatives the reservoir simulation results of a reservoir simulator.

Therefore , when the production $d_{obs}$ is given, the probability distribution function of the reservoir parameters $m$ is defined as:

$$\begin{aligned} & f(m|d_{\text{obs}}) \propto f(d_{\text{obs}}|m) f(m) \\ & \propto \exp[-\tfrac{1}{2}(d_{\text{obs}} - g(m))^T C_D^{-1}(d_{\text{obs}} - g(m)) - \tfrac{1}{2}(m - m_{\text{pr}})^T C_M^{-1}(m - m_{\text{pr}})] \\ & \propto \exp[-O(m)] \end{aligned} \tag{4}$$

The goal of the inversion is to search for the optimal parameters $m$ which can make the probability calculated by Eq. (4) maximum, and the Eq. (4) can be simplified as:

$$O(m) = \frac{1}{2}(m - m_{\text{pr}})^T C_M^{-1}(m - m_{\text{pr}}) + \frac{1}{2}(d_{\text{obs}} - g(m))^T C_D^{-1}(d_{\text{obs}} - g(m)) \tag{5}$$

Then, Eq. (5) represents the inversion objective function which requires both the difference of the updated model and the prior mean model and the disparity of the simulated production and the actual production are minimal and the optimal reservoir parameters are obtained when the objective function value is minimal.

### 2.2. Preparation of inversion parameters and indexes

Generally, a fracture in a two-dimensional reservoir can be simply abstracted as a one-dimensional line segment which can be described by midpoint coordinate $(x_0, y_0)(m)$; azimuth $\alpha$, (°); extension length $L$ ($m$) and opening $H$ ($mm$). The azimuth $\alpha$ is the angle which rotates clockwise from the $x$ axis to the fracture, and its range is $-90^{\circ} \le \alpha \le 90^{\circ}$ or $0^{\circ} \le \alpha < 180^{\circ}$ (Dong-Liang, 2007; Jian et al., 2011), as shown in Fig. 1.

Considering the complexity of the inverse problem, the opening of a fracture is not chosen as inversion parameter in this paper. Because the midpoint coordinate $(x_0, y_0)$, azimuth angle $\alpha$ and extension length $L$ of a fracture directly determine the shape and distribution of fractures and then determine the effect of fractures on the flow behavior, which means that production performance is highly sensitive to those parameters. Therefore, the midpoint coordinate $(x_0, y_0)$, azimuth angle $\alpha$ and extension length $L$ of fractures are chosen as inversion parameters for the inversion of fractures.

Production information such as field oil production rate (FOPR) and field water production rate (FWPR) as well as the well oil

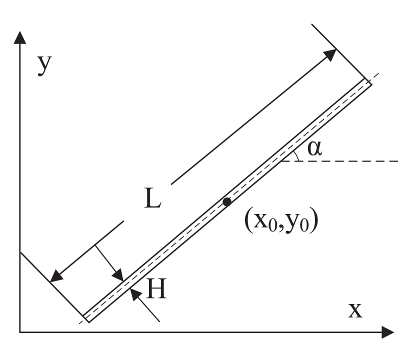


**Fig. 1.** The chart of a fracture in a two-dimensional reservoir.