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Facies proportion uncertainty in presence of a trend

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ABSTRACT

Categorical variable modeling is of great significance for resource estimation as it defines a major aspect of geological heterogeneity and uncertainty. Stochastic simulation can be used to generate multiple equally-probable realizations that describe the uncertainty in the spatial distribution of categorical variables such as facies. These realizations reasonably sample the space of uncertainty if the input statistics are well known. However, the input statistics are often poorly defined due to sparse data. The prior parameter uncertainty related to proportions of different categories is required to achieve an accurate evaluation of the space of uncertainty. For decision making and risk analysis, it is critical to have an accurate and precise model of uncertainty associated with 3924 the subsurface geology. A methodology is proposed, implemented and checked to quantify parameter uncertainty related to facies proportions in presence of a locally varying proportion model (a trend model). Unconditional sequential indicator simulation (SIS) is employed to implement the spatial bootstrap and quantify the prior proportion uncertainty. A trend building algorithm provides multiple realizations of the trend model based on the spatial bootstrap realizations of sampled data. Passing this prior parameter uncertainty through geostatistical simulation provides a realistic posterior model of uncertainty that accounts for the data configuration, conditioning, spatial correlation, and the domain limits.

1. Introduction

An accurate assessment of geological uncertainty plays a key role in resource estimation and risk management (Hanea et al., 2015; Bratvold and Begg et al., 2006). The spatial distribution of facies defines the stationary domains for continuous properties such as porosity and permeability and explains a major aspect of spatial heterogeneity and geological uncertainty (Pyrcz and Deutsch, 2014; Falivene et al., 2006). The ensemble of facies and property realizations can quantify a realistic space of uncertainty if the input statistical parameters are well known. However, limited well data does not permit unambiguous specification of the required parameters (Dowd and Pardo-Igúzquiza, 2002; Babak and Deutsch, 2009). Thus, the uncertainty represented by geostatistical realizations based on the same underlying statistical and geological parameters is too small (Khan and Deutsch, 2015; Wang and Wall, 2003). The uncertainty associated with the input statistical parameters is referred to as parameter uncertainty and must be integrated into the geostatistical modeling to obtain an accurate model of uncertainty. There is a significant body of literature on accounting for parameter uncertainty in geomodeling (Babak and Deutsch, 2009; Kitanidis, 1986; Christakos and Li, 1998; Wang and Wall, 2003). This paper is focused on developing a practical and effective methodology for integration of proportion uncertainty into geostatistical simulation of categorical variables. Khan and Deutsch (2015) proposed a two-step methodology for integration of parameter uncertainty in geostatistical modeling of continuous variables. The first step is based on quantifying the prior parameter uncertainty in input statistics using the spatial bootstrap (Solow, 1985). This technique accounts for the configuration and the spatial correlation between sampled data. The second step transfers the prior parameter uncertainty through the geostatistical modeling process that narrows the uncertainty by considering the effect of conditioning data and accounting for the finite domain limits. In this paper, this methodology is implemented to quantify the proportion uncertainty for facies in presence of trends. A trend is a locally varying model of facies proportions that is inferred based on available data and characterizes the local geological framework (Babak et al., 2014). Categorical variables are often modeled with a trend and the uncertainty of proportions is of great significance for resource estimation. A geostatistical work flow is presented here to obtain an accurate estimation of uncertainty in proportions of categorical variables. The proposed work flow is checked by an experimental setup and its implementation is studied through a case study using a synthetic data set generated based on real well data.

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The spatial bootstrap for categorical variables is implemented using unconditional sequential indicator simulation (SIS) at data locations. SIS is a well-established stochastic technique for categorical variable modeling (Journel and Isaaks, 1984; Journel, 1983; Journel and Alabert, 1989). There are some concerns about indicator simulation. The indicator variograms only consider two-point statistical measures and there is no explicit control over the cross correlation between the simulated categories (Deutsch, 2006). However, there are some good features of SIS that makes it a reasonable technique for the spatial bootstrap. For one thing, the required modeling parameters can be easily inferred from limited well data. Indicator kriging is a robust algorithm to quantify local conditional probabilities and the sequential simulation provides a straightforward way to transfer uncertainty in spatial distribution of categorical variables (Deutsch, 2006). The prior proportion uncertainty is quantified by unconditional SIS and represented by multiple realizations of the trend model that can be integrated in different facies modeling technique including truncated Gaussian (TG), truncated pluri-Gaussian (TPG), multiple point statistics (MPS) and object based algorithms (Matheron et al., ; Armstrong et al., 2003; Strebelle, 2002; Deutsch and Wang, 1996).

2. Methodology

Integrating parameter uncertainty related to input statistical parameters is necessary to obtain a realistic model of geological uncertainty. Fixing input parameters and running multiple geostatistical realizations leads to an unrealistically low uncertainty in global resources or resources over larger volumes of interest for production. For facies modeling, the proportions are the main input statistical parameter. Thus, parameter uncertainty associated with facies proportions is referred to as proportion uncertainty. Facies proportions often follow a trend that is inferred based on available data and reveals the characteristics of the local geological framework. In order to quantify the prior uncertainty in proportions of facies based on limited sampled data, statistical resampling or the spatial bootstrap is employed. This is done by unconditional sequential indicator simulation at the data locations where the base-case trend model is considered in the simulation process. In this context, a trend building algorithm is used to generate the base-case trend based on available facies data. The trend building algorithm is also used to generate realizations of the trend model based on the spatial bootstrap realizations of facies. The prior uncertainty in facies proportion is then represented by multiple realizations of the trend model that account for the configuration and the spatial correlation of the data. Integrating this prior parameter uncertainty into the geostatistical modeling work flow narrows it by considering the effect of the conditioning data and accounting for the finite domain limits. In this context, each realization of the trend model is used as the input statistics for geostatistical modeling and generating the corresponding spatial distribution of facies within the domain of interest. This results in a realistic posterior uncertainty that accounts for parameter uncertainty associated with facies proportions. The proposed methodology was checked to assure that the modeled posterior uncertainty is realistic. Fig. 1 shows a flow chart that summarizes the workflow of the proposed methodology.

2.1. Unconditional indicator simulation at data locations

The spatial bootstrap sampling is implemented by unconditional indicator simulation. This provides the ability of considering large data sets with high resolution sampling down wells. Consider a regionalized, random categorical variable $Z(\mathbf{u})$ at location $\mathbf{u}$ within a stationary domain A and with K different categories or facies. The categories are mutually exclusive meaning that at each location only one category exists. Also, one of the categories must exist at all locations. In this context, categorical variables can be expressed as a series of indicator variables:

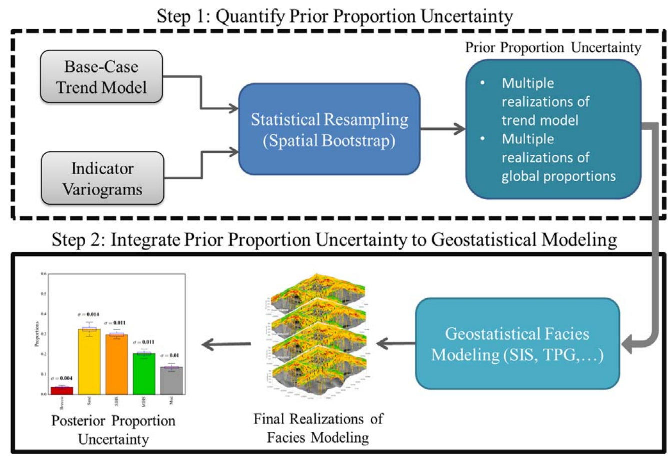

Fig. 1. A flow chart to explain the proposed methodology for quantification and integration of proportion uncertainty.

$$I(\mathbf{u}, k) = \begin{cases} 1, & \text{if} Z(\mathbf{u}) = k \\ 0, & \text{otherwise} \end{cases}, \quad k = 1, \ldots, K, \quad \forall\ \mathbf{u} \in A \tag{1}$$

An indicator variable can be interpreted as the binary probability of a category to prevail at a particular location: the probability is 1 if it prevails and 0 if it does not (Deutsch, 2006). Indicator variography is used to quantify the transition probability for each category as a function of vector Euclidean distance. This quantifies the spatial continuity for each of the K indicator variables. The indicator variogram is calculated, interpreted and modeled to represent the two-point statistical spatial variability and is expressed as:

$$\gamma_I(\mathbf{h}; k) = \frac{1}{2}E\{[I(\mathbf{u} + \mathbf{h}; k) - I(\mathbf{u}; k)]^2\}, \quad k = 1, \ldots, K \tag{2}$$

where $E\{\}$ is the expected value or probability weighted average and $\mathbf{h}$ is the distance vector between the two points. Indicator kriging is used to estimate the local distribution of uncertainty conditioned to local indicator data. In presence of n nearby local data, the kriging estimator is written:

$$I^*(\mathbf{u}; k) - P(\mathbf{u}; k) = \sum_{\alpha=1}^{n} \lambda_\alpha(\mathbf{u}; k)[I(\mathbf{u}_\alpha; k) - P(\mathbf{u}_\alpha; k)], \quad k = 1, \ldots, K \tag{3}$$

where P($\mathbf{u}$;k) is the prior model of local proportions provided by the trend model that is inferred based on local data and represents the subsurface geological formation. The kriging weights are assigned to minimize the error variance providing the following system of linear equations:

$$\sum_{\beta=1}^{n} \lambda_\beta(\mathbf{u}; k)C(\mathbf{u}_\alpha, \mathbf{u}_\beta; k) = C(\mathbf{u}_\alpha, \mathbf{u}; k), \quad \alpha = 1, \ldots, n \quad \&k = 1, \ldots, K \tag{4}$$

$$C(\mathbf{h}; k) = C(0, k) - \gamma_I(\mathbf{h}; k)$$

where C($\mathbf{h}$;k) is the covariance function calculated based on the corresponding indicator variogram (Journel, 1983). In this context, $C(0, k)$ denotes the indicator variance. The estimator of indicator kriging for each category provides the local conditional probability/proportion at an unsampled location that can be integrated to infer a conditional cumulative distribution function (CCDF). These estimated probabilities are post processed to ensure they are within 0–1 and that they sum to 1. Indicator simulation is then implemented by Monte Carlo sampling of the CCDF. Fig. 2 summarizes how a CCDF is formed and used for simulation based on Monte Carlo sampling.

For the spatial bootstrap, all the data locations are considered as unsampled locations and simulated in a random sequence. In this context, the simulation is implemented by inferring the CCDF based on