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Determination of oil reservoir permeability from resistivity measurement: A parabolic identification problem

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ABSTRACT

In this paper we propose a new method in order to determine the oil reservoir permeability starting from the experimental values of the radial resistivity variation caused by mud filtrate invasion process. This process consists of two coupled phenomena: the filter cake formation (modeled by a Cauchy problem for an ordinary differential equation of first order) and the mud filtrate diffusion process through the porous media (modeled by a mixed problem for diffusion equation). The oil reservoir permeability is determined by the identification method for a parabolic problem. In this context we obtain two algorithms for the numerical calculus of oil reservoir permeability. These algorithms use as the main tool the derivative of the objective functional with respect to the reservoir permeability. The method for the reservoir permeability determination presented in this paper was numerically tested using four sets of data (which correspond to four wells). The oil reservoir permeability values obtained with our algorithms are in good agreement with the experimental data.

1. Introduction

Permeability is one of the most important properties of the porous medium and it represents the medium's capacity of allowing the flow of fluids determined by a pressure gradient. The permeability depends primarily on the structure of the porous medium. The structure of the porous medium has a very large variation, from an ideal structure (rectilinear capillary tubes with constant section or spherical granule with constant diameter and cubical packing) to a very complex one. In most cases the oil reservoirs have a complicated structure of the porous medium, with a random pore distribution. This distribution forms capillary networks with various diameters and degrees of interconnection. The roughness and tortuosity of capillary networks influence the fluids flow. Capillary tubes' size and shape are determined by many factors such as: lithology, porosity, the degree of cementation, the degree of cracking, the solubility of the mineral skeleton, the clay volume. All these factors lead to a large range of variation of rock collectors' permeability from $10^{-16}\ m^2$ to $10^{-12}\ m^2$.

There are three methods for oil reservoir permeability determination: (Mohaghegh et al., 1996):

a) laboratory measurements (core analysis);
b) well test interpretation;
c) well logging interpretation.

The methods (b) and (c) are empirical (empirical correlations) (Fertl, 1987; Balan et al., 1995), statistical (multiple variable regression) (Yao and Holditch, 1993; Habibi et al., 2014), those based on "virtual measurement methods" (artificial neural networks) (Balan et al., 1995; Mohaghegh et al., 1996; Mohaghegh, 2000; Huang et al., 1996) (genetic algorithm) (Kaydani and Mohebbi, 2013; Kaydani et al., 2011), (fuzzy logic modeling) (Abdullraheem et al., 2007; Kaydani et al., 2012) and those who use the inverse method.

In empirical methods permeability is achieved by correlation with other petrophysical measurable properties: porosity, specific area, irreducible water saturation, gradient resistivity.

The methods that use well logging interpretation are based on the phenomenon of mud filtrate invasion. As a result of this process mud filtrate and solid particles (from the mud and from the dislocated rock) enter the reservoir. As a result mud cake of some thickness is formed on the wall of the well and the invaded (damaged) zone appears. The mud filtrate that enters the rock collector pushes away a part of the existent fluids from the porous space and modifies the water saturation and oil saturation of the reservoir. These changes have as a result the radial resistivity variation, radial water saturation and the collector permeability radial variation (Iscan et al., 2007; Civan, 1994). Therefore the mud filtrate invasion leads to a continuously variable permeability in radial direction, from a very small value (the mud cake permeability) to a value

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Nomenclature

A	Archie's tortuosity/cementation factor
C	mud filtrate concentration in formation (kg/m^3)
C_f	mud filtrate concentration in drilling fluid (kg/m^3)
C_s	solid particle concentration in drilling mud (kg/m^3)
C_{ref}	reference mud filtrate concentration (kg/ m^3)
D	diffusion coefficient (m^2/s)
g	gravitational acceleration (m/s^2)
h	formation thickness (m)
J	the objective function
k	formation permeability (m^2)
k_c	cake permeability (m^2)
k_m	minimum value of controls
k_M	maximum value of controls
K_{ad}	the set of admissible controls
L	the Lagrange function
p	Lagrange multiplier for restriction (39).
p_1	Lagrange multiplier for restriction (41).
p_2	Lagrange multiplier for restriction (42).
Pe	Peclet number (dimensionless)
Q_{CEC}	Cation-exchange capacity
q	volumetric flow rate (m^3/s)
m	Archie's cementation exponent
n	Archie's saturation exponent
r	radial coordinate (m)
r_e	drainage radius (m)
r_w	well radius (m)
R_t	true resistivity (Ohm·m)
R_w	water resistivity (Ohm·m)
R_{sh}	shale resistivity (Ohm·m)
S_{or}	residual oil saturation (fraction)
S_{wir}	irreducible water saturation (fraction)
S_w	water saturation (fraction)
t	time (seconds)
t_{inv}	invasion time (seconds)
u	the filtration velocity (m/s)
V_{sh}	shale volume
x_c	mud cake thickness (m)
α	empirical parameter (dimensionless)
α_1	coefficient in irreducible water saturation formula
α_2	coefficient in mud cake thickness equation
β	empirical parameter (dimensionless)
Δp	overbalance pressure (Pa)
μ_f	mud filtrate viscosity (Pa·s)
Φ	formation porosity (fraction)
Φ_c	mud cake porosity (fraction)
ρ_c	mud cake density (kg/m^3)
ψ	the sensitivity function
ε	real number

corresponding to uncontaminated zone (the reservoir permeability).

In order to determine the mud filtrate concentration in the invaded zone we use mud filtrate invasion models. There are two main methods for the study of mud filtrate invasion (Civan, 2007):

a) single-phase mud filtrate invasion model (mixed problem for diffusion equation) (Civan and Engler, 1994; Parn-anurak and Engler, 2005; Windarto et al., 2012);
b) two-phase wellbore mud invasion (Civan, 1994; Ramakrishnan and Wilkinson, 1997).

A frequently used method for determining the oil reservoir permeability is the inverse method. In this method we begin from some experimental values of some physical quantities, determined in the contaminated zone, from which we determine the reservoir permeability.

Corresponding to the method used for the study of mud filtrate invasion there are inverse methods that use the two phase model and methods that use the single-phase model (the diffusion equation).

The analysis of some inverse problems that use the two phase model of mud filtrate invasion is made in Ramakrishnan and Wilkinson (1997), Phelps et al. (1984), Ramakrishnan and Wilkinson (1996), Semmelbeck et al. (1995), Alpak et al. (2008).

Gottlieb and Dietrich (1995) solve an inverse problem for a parabolic differential operator with several right-hand sides in order to determine a spatially varying permeability coefficient (distributed parameter identification problem). They use the least-squares method and Tikhonov regularization, as in the papers of Chavent et al. (1975) and Kravaris and Seinfeld (1985).

In this paper a new method is presented in order to determine the oil reservoir permeability starting from experimental values of radial resistivity variation (caused by mud filtrate invasion). In order to determine the mud cake thickness and the mud filtrate concentration in the invaded zone, we use the models presented in Windarto et al. (2012). We determine the reservoir permeability by using an inverse method for parabolic problems. We use here the identification method which gives us the desired value of the coefficient control (the reservoir permeability) by minimizing a given objective functional.

The structure of the paper is the following: in section two we present some remarks regarding the correlation between the reservoir electrical resistivity and the water saturation (Archie's relation). In section three we briefly present the method used in Windarto et al. (2012) in order to determine the mud cake thickness and the mud filtrate concentration in invaded (damaged) zone. Section four contains the main result of the paper: the algorithms for oil reservoir permeability determination. The main tool used here is the derivative of the objective functional with respect to the control coefficient; we obtain the optimum control by using the bisection method (Hendrix and Tóth, 2010). We make here the assumption that the objective functional has a single critical point, which is a global minimum in the interval of variation of the control coefficient. Section five contains numerical results that prove the effectiveness of the algorithms presented in section three. The appendix contains the formal deduction of the sensitive function's properties, of the adjoint system and of the derivative of the objective functional with respect to the control coefficient.

2. The resistivity-water saturation correlation

A rock collector can be fully or partially saturated with water, the rest being occupied by oil and gases. A rock without clay content has the real resistivity R_t greater than a rock saturated with water. The correlation between water saturation and the resistivity for a rock without clay content it is give by Archie's relation: