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Double circular arc model based on average shear stress yield criterion and its application in the corroded pipe burst

Zhanfeng Chen[a], Sunting Yan[a], Hao Ye[a], Zhengzhi Deng[b], Xiaoli Shen[a], Zhijiang Jin[a,*]

[a] *Institute of Process Equipment, College of Energy Engineering, Zhejiang University, Hangzhou 310027, PR China*
[b] *Ocean College, Zhejiang University, Hangzhou 310027, PR China*

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ABSTRACT

Corrosion is one of the major causes of damage in offshore and onshore pipes. Based on the double circular arc (DCA) model and the average shear stress yield (ASSY) criterion for isotropic hardening materials, a new burst pressure equation for corroded pipes is developed in this paper. The ASSY criterion is a weighted average of the maximum shear stress and the octahedral shear stress and it is adopted to predict the burst pressure of the pipelines with corrosion defects. The accuracy of our predictive equations is validated respectively by both the ideal and corroded pipes. Firstly, the burst pressure equation of a corroded pipe is degraded into an ideal pipe's. Then we compare the calculated results with the full-scale experimental data. The calculated results are consistent with the experimental data for the ideal pipes. Again, we make a comparison of the predicted burst pressures of corroded pipes with the ones from the elastic-plastic finite element method (FEM) and the experiment. The calculated results show a good agreement with the numerical calculations and experimental data for the corroded pipes. Our research reveals that the corrosion defects can decrease the burst pressure for a pipe; the yield criterion and the strain hardening exponent are crucial to accurately capture the burst pressure. Our research is beneficial to evaluate the integrity of the corroded pipes.

1. Introduction

Pipelines are recognized as crucial equipment in the transporting gas, oil and other hydrocarbons. Because of the existence of corrosive substances, the pipes are often corroded. The corrosion will reduce the burst pressure of the pipes. It is critical for the pipes to capture the burst pressure in the engineering design and integrity assessment. Therefore, it is indispensable to predict the burst pressure of the pipes with corrosion defects.

The burst pressure is usually defined as the limit load-bearing capacity of a pipe under the internal pressure. The predictive models for burst pressure of the pipes have been developing for more than one half century (Zhu and Leis, 2012). A host of theoretical, numerical and experimental researches done are to develop quite a few analytical and empirical equations to predict the burst pressure. As we know, the yield criterion is a key factor for the pipe burst, and it also exerts a tremendous of fascination to us. Therefore, we review the main work from that facet. From the point of view of the Tresca yield criterion, the earliest theoretical model tracks back to Turner (1910). He proposed a burst pressure equation based on the Tresca yield criterion and thin-walled tube theory. Bailey (1930) presented another burst pressure equation containing the strain hardening exponent. American Society of Mechanical Engineers (1962) gave two different burst pressure equations according to the ratio of the radii. DNV (1999) introduced the flow strength into the burst pressure equation. Fletcher (2003) developed a burst pressure equation based on the flow strength and ultimate strain. Chen et al. (2015a) presented a new burst pressure equation for the worn pipes based on the stress function method and thick-walled tube theory. In the terms of the von Mises yield criterion, Nadai (1931) developed the first burst pressure equation. Afterwards, Nádai (1950) proposed another burst pressure equation containing the strain hardening exponent. Faupel (1956) poured attention into the yield-to-tensile ratio and provided a burst pressure equation with the yield-to-tensile ratio. To predict the burst pressure, Marin and Sharma (1958) proposed three different equations for thin-walled tubes. Chen et al. (2015b) respectively provided two burst pressure equations for the restrained and end-capped pipes. Recently, Klever (1992), Stewart et al. (1994) and Klever and Stewart (1998) pointed that the experimental data of the burst pressure are located between the results of the Tresca criterion and those of the von Mises criterion and provided a burst pressure equation. Later, Zhu and Leis (2004, 2006, 2007) found that the Tresca predictions were the lower

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* Corresponding author.
E-mail address: jzj@zju.edu.cn (Z. Jin).

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Nomenclature

c_0, c_1, c_2	coefficients in Eq. (18)
D, D_i, D_m	external diameter, internal diameter and mean diameter, respectively
e	2.718, the natural logarithm base
f_0, f_1, f_2	coefficients in Eq. (18)
g_0, g_1, g_2	coefficients in Eq. (25)
h_0, h_1	coefficients in Eq. (25)
k	$\frac{D}{D_i}$
l_0, l_1, l_2	coefficients in Eq. (26)
n	strain hardening exponent: $n = \exp(1 + \varepsilon_{\text{uts}})$, where ultimate strain ε_{uts}corresponds to ultimate tensile strength σ_{uts}
p_b, $p_b^{\text{prediction}}$, $p_b^{\text{expriment}}$	burst pressure, burst pressure of present method and burst pressure of experiment, respectively
R_i	inner radius of a pipe
RE	relative errors
s_0, s_1, s_2	coefficients in Eq. (26)
t, $t_{\max}$, $t_{\min}$	wall thickness, maximum wall thickness and minimum wall thickness, respectively
x, y	components of the Cartesian coordinate system
Y/T	yield-to-tensile ratio
χ	eccentricity ratio δ/t
ε_{uts}, ε_{true}	uniform strain and true strain, respectively
$\tau_{\max}$, τ_{M}, τ_{A}	the maximum shear stress, the von Mises effective shear stress and the average shear stress, respectively
δ	distance between the centres of inner and outside circles
ξ	corrosion ratio $2\delta/t$
λ	ratio of thickness to diameter t/D
μ	a stress parameter introduced by Lode
σ_o, σ_{flow}, σ_{uts}, σ_{true}	the yield strength, flow strength, ultimate tensile strength (UTS) and true stress, respectively
σ_1, σ_2, σ_3	the first, second and third principal stresses, respectively
σ_{T}, σ_{M}, σ_{A}	the Tresca equivalent stress, the von Mises equivalent stress and the ASSY effective stress, respectively

boundary and the von Mises predictions were the upper boundary for the burst pressure and developed a new burst pressure equation based on the average shear stress yield (ASSY) criterion and the thin-walled tube theory.

The corrosion of pipes is increasingly serious. It is indispensable to develop a burst pressure for the corroded pipes. To further explore the reliable and accurate burst pressure equation for the thick-walled pipes, we establish a new double circular arc (DCA) model and develop a new burst pressure equation for the corroded pipes based on the ASSY criterion. The accuracy of the calculated results is validated by the comparison with the experimental data and finite element method (FEM). The new burst equations will extend the application scope of the ASSY criterion and provide a new method to assess the integrity of the pipes.

2. Geometric model of a corroded pipeline

Pipelines with a corrosion defect are usually simplified as Fig. 1(a). For more simply describing by using mathematical language, we established the DCA model constituted by two eccentric circles, as shown in Fig. 1(b). In the DCA model, the dashed circle represents the initial inner wall of the pipe without corrosion defects. The dashed circle shares the same centre O_0 with the outer wall of the pipe. The inner circle centre with corrosion defects is O_1. The initial wall thickness of the pipe A_0B_0 is also the maximum wall thickness of the pipe with corrosion defects. The minimum wall of the corroded pipes is A_1B_0. As a plane strain problem, corroded status of the pipeline can be described by using the parameter χ:

$$\chi = \frac{t_{\max} - t_{\min}}{t_{\max} + t_{\min}}, \tag{1}$$

or

$$\chi = \frac{\delta}{t}, \tag{2}$$

where $t_{\max}$ and $t_{\min}$ denote the maximum and minimum thicknesses of the pipelines with corrosion defect, respectively; $\delta = (t_{\max} - t_{\min})/2 = O_0O_1$ denotes the distance between the inner and outer circle of the pipelines with corrosion defects; $t = (t_{\max} + t_{\min})/2$ denotes the thickness of the pipe without corrosion defects.

3. Multiaxial yield criteria of a corroded pipeline

3.1. Three classical yield criteria

3.1.1. The Tresca criterion

The Tresca criterion is a classical yield criterion in the strength theory for isotropic ductile materials (such as mild steel and alloy steel). This yield criterion, known as the maximum shear stress criterion, has a simple form and wide application in plastic materials. Because of the neglect of the second principal stress, the calculated results usually seem a little conservative. The Tresca criterion can be expressed as

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_o}{2}, \tag{3}$$

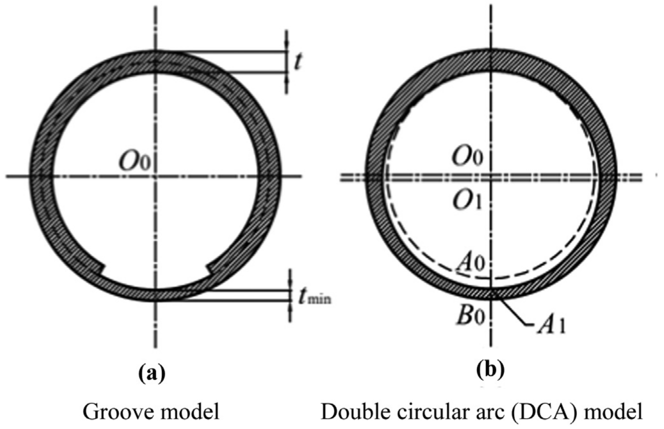
where $\tau_{\max}$ is the maximum shear stress, σ_1, σ_3 are the first and third principal stresses ($\sigma_1 \geq \sigma_3$); σ_y is the yield stress in tension.

For convenience, the Tresca equivalent stress, σ_{T}, is defined as

$$\sigma_{\text{T}} = \sigma_1 - \sigma_3 = \sigma_o, \tag{4}$$

3.1.2. The von Mises criterion

The von Mises criterion is another classical yield criterion in the strength theory. This yield criterion is often referred to as distortion energy yield criterion. This yield criterion is that the material yield is the result of the distortion energy density. The von Mises criterion can



(a) Groove model (b) Double circular arc (DCA) model

Fig. 1. Geometric models of the corroded pipes (a) Groove model (b) Double circular arc (DCA) model.