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Peripheral water injection efficiency for material balance applications

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A R T I C L E I N F O

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A B S T R A C T

The simple approach consisting of including injected water directly into the reservoir material balance equations is not appropriate to model peripheral injection, as it does not account for water lost to the aquifer as well as time required for pressure to diffuse to the reservoir boundary. Based on this observation, the authors have extended the van Everdingen and Hurst (1949) unsteady state edge aquifer model to account for peripheral sources.

Taking advantage of the pressure diffusion equation linearity and problem symmetries, for simplified circular and linear geometries the problem can be cast as one-dimensional regardless of the number and position of peripheral injectors. Solutions are reported in the form of a tabulated cumulative efficiency function, defined as the amount of water having reached the reservoir boundary owing to the presence of a source injecting at unit rate, vs. time. Superposition principle can then be used to calculate time-dependent water influx for an arbitrary number of injectors and injection history.

Solving the two-dimensional problem further provides the lateral influx distribution, and shows that pressure support efficiency as defined in this work, relevant to material balance applications, is conceptually different from transport efficiency provided by streamlines analysis. The latter is indeed unable to single out the individual contribution of a specific injector to reservoir voidage replacement from that of its neighbors and the aquifer itself.

Nomenclature

Symbol	Dimension	Description
ϕ		Aquifer porosity
k	length^2	Aquifer permeability
c_t	pressure^{-1}	Total aquifer compressibility
h	length	Aquifer thickness
x_a	length	Linear aquifer length
w_o	length	Linear aquifer width
x_{inj}	length	Injection position in linear geometry
r_o	length	Circular reservoir radius
r_a	length	Circular aquifer radius
θ_o		Circular aquifer encroachement angle
r_{inj}	length	Injection position in circular geometry
U_L,U_C	length^3/ pressure	Linear, circular aquifer constants (Eqs. (7) and (12))
t	time	Time
ψ	pressure	Water potential (Eq. (3)), simply referred to as "pressure"
ψ_{init}	pressure	Initial aquifer potential
ψ_o	pressure	Reservoir boundary potential
$\mathcal{P}$	pressure	Reference pressure
q_e	length^3/ time	Reservoir influx rate
W_e	length^3	Reservoir cumulative influx
$\mathcal{W}_L$, $\mathcal{W}_C$		Linear, circular cumulative aquifer rate functions (Eqs. (6) and (11))
$\mathcal{Q}_L$, $\mathcal{Q}_C$		Linear, circular instantaneous aquifer rate functions (Eq. (18))
q_{inj}	length^3/ time	Injection rate (at subsurface conditions)
W_{inj}	length^3	Cumulative injection (at subsurface conditions)
$\mathcal{E}_L$, $\mathcal{E}_C$		Linear, circular cumulative injection efficiency functions (Eqs. (32) and (33))
$\mathcal{F}_L$, $\mathcal{F}_C$		Linear, circular instantaneous injection efficiency functions (Eq. (34))
ψ^*		Potential nondimensionalized by $\mathcal{P}$
property$^*_L$		Nondimentionalized property (length by x_a, time by τ_L (Eq. (8)))
property$^*_C$		Nondimentionalized property (length by r_o, time by τ_C (Eq. (13)))

1. Introduction

Flow in a petroleum reservoir is essentially inertia-free (i.e., pressure, capillary, and gravity forces are in equilibrium with viscous

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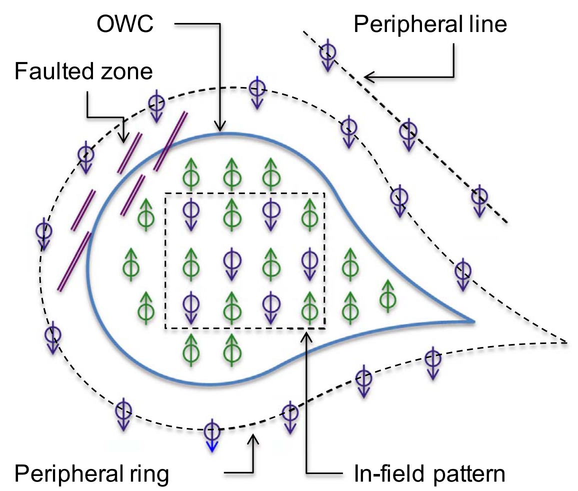
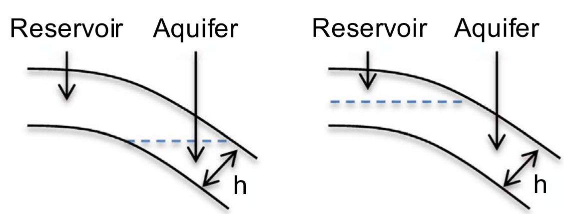
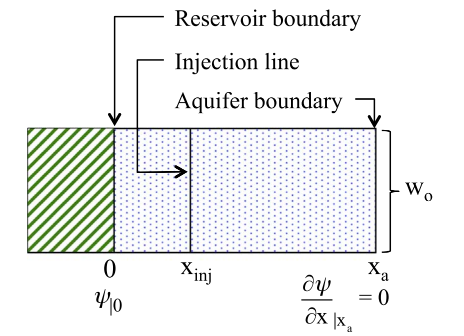


Fig. 1. Areal view of a reservoir with edge aquifer drive, produced in secondary mode through both peripheral and pattern injection. Intuitively, it is expected that the peripheral injection line and the western part of the injection ring will have a low efficiency due to their distance and the presence of sealing faults, respectively. The meaning of "low" will be detailed in the paper.

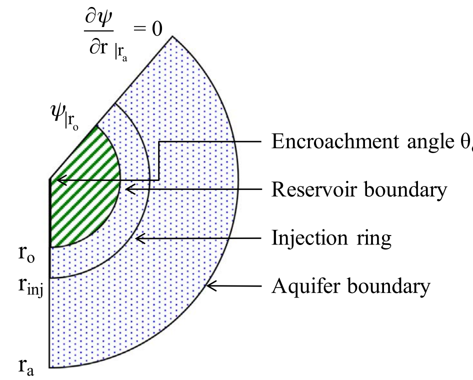


(a) Edge Aquifer. (b) Bottom Aquifer.

Fig. 2. Sketches of edge (a) and bottom (b) reservoir-aquifer sections, after Coats (1962). We will see that the arbitrary separation between reservoir and aquifer (horizontal dashed line), typically taken as the oil-water contact in the literature treating aquifer influx, has a strong impact on the calculation of peripheral injection efficiency. Only edge aquifers are considered in this paper.



(a) Linear geometry.



(b) Circular geometry.

Fig. 3. Areal view of linear (a) and circular (b) aquifer geometries. The constant "terminal pressure" aquifer problem disregards injection, and is obtained using a constant reservoir edge pressure different from the initial aquifer pressure. The "peripheral injection" problem is obtained using a constant reservoir edge pressure equal to the initial aquifer pressure, accounting for an injection line or ring. In both cases, no flow is allowed at the outer boundary. The total influx is the sum of the solutions of these two independent problems.

forces (Bear, 1972)), hence is governed by conservation of mass; this is the basis of modern reservoir simulation (Aziz and Settari, 1979), as well as more classic but still of paramount importance material balance calculations (Schilthuis, 1936; Dake, 1978; Wang et al., 1992; Petroleum Experts, 2012).

In the latter, the reservoir (see the schematic illustration in Fig. 1) is approximated by a single tank with uniform pressure and oil/gas compositions. By relating the difference between produced and injected volumes to changes in pressure, it is possible to estimate properties such as original oil in place or gas cap size, as well as the aquifer strength; material balance can also be used to predict future pressure and primary production, especially for gas reservoirs.

Single-tank calculations apply provided the reservoir is well connected throughout, and the characteristic time of pressure variation (e.g., duration of primary depletion) is longer than the characteristic time of pressure diffusion between wells. For large reservoirs, the latter condition typically requires drainage and injection points to be evenly distributed. Aquifers are not included in the "reservoir" definition, which is limited to the hydrocarbon-bearing volume of rock. They can indeed be of large extent and only communicate with the reservoir through a limited surface (edge or bottom, see Fig. 2); their response to variations of reservoir pressure is therefore not instantaneous, and different models have been developed to approximate such response.

These can be separated in two categories: unsteady state (USS) and pseudo-steady state (PSS). USS models provide a solution to the full problem of pressure diffusion in the aquifer, typically considering idealized circular or linear geometries. The advantage is that for edge aquifers considered in this paper, if the system thickness "h" is small

Table 1
First three solutions to Eq. (15).

r_a^*	a_1	a_2	a_3
1.2	7.5667	23.469	39.214
1.5	2.8899	9.3448	15.660
2.0	1.3608	4.6459	7.8142
3.0	0.6256	2.3040	3.8954
4.0	0.3935	1.5266	2.5908
5.0	0.2824	1.1392	1.9392
6.0	0.2181	0.9075	1.5486
7.0	0.1765	0.7534	1.2884
8.0	0.1476	0.6437	1.1027
9.0	0.1264	0.5616	0.9636
10.0	0.1103	0.4979	0.8554
20.0	0.0465	0.2318	0.4016
50.0	0.0158	0.0879	0.2189

compared to its extent and the pressure is uniform at the reservoir-aquifer boundary, the problem can be cast as one-dimensional.

Van Everdingen and Hurst (1949) provided semi-analytic USS solutions in circular geometry through the use of Laplace transforms, for the constant "terminal pressure" and "terminal rate" cases (i.e., constant reservoir boundary pressure and constant aquifer influx). These are expressed as infinite summations of exponentially decaying terms involving Bessel functions, for which the authors provide convenient tabulations. Solutions in linear geometry can be obtained from the former in the limit of aquifer radius approaching reservoir radius, or from mathematically analogous problems (Carslaw and Jaeger, 1959); these are then expressed as infinite summations of