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A new capillary pressure model for fractal porous media using percolation theory

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ABSTRACT

The capillary pressure-saturation relation in traditional models is obtained by representing porous medium as a bundle of capillary tubes. The effect of pore connectivity is ignored, which leads to the evident deviation frequently observed between the measured data and model prediction near the breakthrough pressure region. In the present work, a new fractal-percolation-based capillary pressure model is developed. The proposed model accounts for the characteristics of pore size distribution in fractal media, and is based on the percentage of pore volume rather than that of pore number in conventional percolation theory. In the new model, a simple and yet sufficiently accurate relation between the allowed and accessible saturations is applied near the threshold saturation. It results in a high-accuracy prediction of capillary pressure, especially near the breakthrough pressure region. The new model is validated by experimental data. Parameter sensitivity analysis and comparison with the results from conventional fractal theory are presented. This new model is an extension to the conventional fractal theory, as it is consistent when the threshold saturation is equal to zero.

1. Introduction

The capillary pressure in a porous media is the result of the combined effects of the surface and interfacial tensions of the rock and fluids, the pore size and geometry, and the wetting characteristics of the system. The capillary pressure, as a function of saturation, is a fundamental property affecting immiscible multiphase flow in porous media, which makes it of great research significance.

The research on the capillary pressure behavior includes both experimental work and theoretical model development.

Laboratory methods to obtain the relation between capillary pressure and wetting phase saturation in porous media mainly includes mercury intrusion, semipermeable-membrane, self-absorption and centrifugal method. Among these methods, the mercury intrusion is widely used because this method is simple to conduct and rapid. The data can be used to determine the pore size distribution, to study the behavior of capillary pressure curves, and to infer characteristics of pore geometry. In addition, O'Meara et al. (1988) showed that mercury injection capillary pressure data of water-oil systems (normalized using Leverett J-function) are in good agreement with the strongly water-wet capillary pressure curves obtained by other methods. Brown (1951) found that gas-oil capillary pressure data can be made to agree with mercury injection capillary pressure data by using an appropriate scaling factor.

For theoretical model development, the capillary pressure model is established mainly through describing the pore structure characteristics of porous media by the simplified capillary bundle model. Researchers use various techniques to represent the pore structure of natural formation, including sandstone, carbonate, shale, coal, etc. (Wong et al., 1986; Friesen and Mikula, 1987; Krohn, 1988; Angulo and Gonzalez, 1992; Zhang et al., 2014; Naik et al., 2015; Xiong et al., 2015; Zhou and Kang, 2016). The experimental results show that, the pore space of porous media formed with randomly stacked natural particles consists with fractal characteristic (Pfeifer and Avnir, 1983; Katz and Thompson, 1985; Hansen and Skjeltorp, 1988; Bu et al., 2015; Lai and Wang, 2015; Sakhaee-Pour and Li, 2016).

Hunt (2004) suggested that fractal concepts might be well suited to model fluid movement in a porous medium, because of their simple descriptions of highly ramified spaces. Different fractal

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capillary pressure models have been developed based on different hypothetical fractal models, including de Gennes (1985), Friesen and Mikula (1987), Tyler and Wheatcraft (1990), Rieu and Sposito (1991), Shen and Li (1994), Perrier et al. (1996), Deinert et al. (2005), Millán and González-Posada (2005), Perfect (2005), Cihan et al. (2007), Li (2010), Liu et al. (2016), and so on. Among these models, the most widely used ones are de Gennes (1985), Friesen and Mikula (1987), and Rieu and Sposito (1991) models. The last one is usually referred to as the RS model. All the three models agree with the measured data very well, except for the breakthrough pressure neighborhood. For drainage-type capillary pressure curve, the region near the breakthrough pressure corresponds to the large pores. Shen and Li (1994) applied fractal model to fit the capillary pressure curve obtained from mercury intrusion method. Their results indicated that traditional fractal model cannot match the trend of capillary pressure-saturation behavior in the breakthrough pressure neighborhood, as stated above.

Based on this phenomenon, Millán and González-Posada (2005) proposed the piecewise fractal model by considering that the capillary pressure curve includes both structural and textural pores and assumed two fractal regimes.

Nonetheless, Larson and Morrow (1981) indicated that the capillary pressure curve depends not only on the geometrical and wetting properties of individual pores but also upon the pores' connections to the surface of the sample. Hunt et al. (2014) also argued that some differences between the modeled and measured results must be due to the fractal models having no percolation concepts built in. Models such as RS (Rieu and Sposito, 1991) are in some sense no different from the capillary bundle model: they do not consider pore connectivity, which is one of the most important characteristics of porous media.

Sahimi et al. (1990) classified models for transport and reaction in porous media as continuum bundle model and discrete percolation network model. The former is not suitable for describing the phenomena in which the connectivity of the pore space or the fracture network is significant. The latter is free of the limitations of the continuum model. It can be applied to describe phenomena from microscopic to macroscopic scales.

Percolation theory is complementary to the network models. The first capillary model using percolation theory was developed by Larson and Morrow (1981), in which the physical concept of accessibility that determines which pores can be invaded by non-wetting phase, and from which pores it can be withdrawn, has been explicitly utilized. Then Heiba et al. (1992) represents a refinement of the model of Larson and Morrow (1981). The basis for Heiba model is that during injection and retraction, the spatial distributions of the pores accessible to and occupied by non-wetting phase, which they refer to as the sub-distributions, are not identical. Consequently, the throat size distribution of the subset of pore space occupied by non-wetting phase differs from the overall throat size distribution. Heiba et al. derived analytical formulae for such sub-distributions. Soll and Celia (1993) defined a set of rules in a network model and developed a computer code (3PSAT) to calculate three-phase capillary-pressure curves. Hunt (2004) presented a capillary pressure model using the percolation theory applied to the fractal of porous media for the first time. The model is different from other models in the following two aspects. First, the particularity of fractal distribution is considered in this model. He presented that using the water content and threshold water content (in the soil field) to replace the allowed fraction p and percolation threshold p_c in conventional percolation theory directly. Second, non-equilibrium process is considered in this model rather than the equilibrium one in conventional percolation model. In this paper he points out that the model can be well applied both in drainage and imbibition of soil porous media.

The new fractal capillary pressure model proposed in this paper integrated all the advantages of the methods above. Conceptually, we follow the model developed by Heiba et al. (1992). The difference lies in that the allowed saturation S_p and threshold saturation S_c is used in this paper, instead of the allowed fraction p and percolation threshold p_c in conventional percolation theory. The saturation is in analog with the pore fraction ranging between zero and one, while applying the saturation avoids the prominent influence of the minimum pore radius, which is difficult to determine and has little effect on macroscopic properties. The equilibrium method is used in this paper. A simple and yet sufficiently accurate expression is employed to calculate the accessible fraction. The saturation is a function of pore distribution. And so the saturation-capillary pressure model of fractal porous media can be obtained. Finally, parameter sensitivity analysis and comparison with the results from conventional fractal theory are performed. The new model is validated by experimental data.

2. The characteristics of fractal porous media

Since the fractal theory was proposed by Mandelbrot (1982), many strict synthetic fractal objects have been proposed by mathematician, which greatly improve our understanding of the fractal media. Among the many synthetic fractal objects, the 2D Sierpinski carpet and its 3D equivalent, the Menger sponge (Fig. 1) have been widely used to model porous materials. The Menger sponge is constructed by starting with a cube of size L (the "initiator length"). For the first iteration, the operational length scale r is (L/b) in which the scaling factor $b = 3$ for the traditional Menger sponge. Divide every face of the cube into b^2 squares, like a Rubik's Cube. This will sub-divide the cube into b^3 smaller cubes. Remove n smaller cubes of size r, leaving $(b^3 - n)$ smaller cubes. Then, repeat the former step for each of the remaining smaller cubes, and continue to iterate ad infinitum. The number of remaining cubes of size r satisfies the following scaling relation (Ghanbarian-Alavijeh et al., 2011):

$$N(=r) = ar^{-D} \tag{1}$$

where a is a constant coefficient, and D is the fractal dimension. Following Friesen and Mikula (1987), let $a = 1$ and obtain the fractal dimension of the sponge as:

$$D = \frac{\log(b^3 - n)}{\log b} \tag{2}$$

The remaining and the removed cubes in Fig. 1 represent solid particles and pores, respectively. Although the pore phase in the sponge is not geometrically fractal, its number-size distribution is given by a power-law function and it has the same fractal dimensionality as the solid phase (Rieu and Perrier, 1997):

$$N(\geq r) = kr^{-D} \tag{3}$$

where $N(\geq r)$ is the number of fractal objects whose size is equal to or greater than r, k is a constant coefficient, and the fractal dimension D typically ranges between 0 and 3 in natural porous materials. So in fact, one fractal dimension scales both solid and pore phases, albeit in different ways.

For natural porous media, it only applies within the specified range, $r_{\min}$ to $r_{\max}$, in which the power-law function (Eq. (3)) is truncated. And natural porous media are more heterogeneous and complex with randomly self-similar instead of the exactly self-similar as shown in Fig. 1.

The probability density function of fractals-the number of