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Review Article

Nonlinearity and solution techniques in reservoir simulation: A review

Pulok Kanti Deb [a], Farhana Akter [a], Syed Ahmad Imtiaz [b], M. Enamul Hossain [c,*]

[a] Department of Process Engineering (Oil & Gas Program), Memorial University of Newfoundland, St. John's A1B 3X5, NL, Canada
[b] Department of Process Engineering, Memorial University of Newfoundland, St. John's A1B 3X5, NL, Canada
[c] Statoil Research Chair in Reservoir Engineering, Department of Process Engineering, Memorial University of Newfoundland, St. John's A1B 3X5, NL, Canada

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A B S T R A C T

Reservoir simulation is used to demonstrate the dynamic physical processes of rocks and fluid properties with high-order nonlinear equations. Currently, different types of simulation models are used in the petroleum industry. These models are solved by numerical techniques to get a solution considering lots of inherent assumption. In this paper, an extensive review is offered on the state-of-the-art literature with a focus on the nonlinearity in partial differential equations related to the petroleum reservoir simulation. A critical analysis is done on the different techniques for solving nonlinear governing equations in a petroleum reservoir. It also addresses the inherent assumptions and properties, the significance of nonlinear solvers and their technical challenges. Finally, the article discusses the impact of the solution of these nonlinearity problems by following the different numerical techniques.

Contents

* Corresponding author.
E-mail addresses: pkd675@mun.ca (P.K. Deb), fakter@mun.ca (F. Akter), simtiaz@mun.ca (S.A. Imtiaz), ehossain@mun.ca (M.E. Hossain).

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1. Introduction

The petroleum industry is the primary key to the global economy, and technological advancement moves forward based on this sector. The energy demand is increasing and currently, the crude oil production is some 90 million barrels per day (EIA, 2016). Extracting more oil and gas out of existing reservoirs is therefore of paramount importance if the industry is to meet the future growth of energy consumption. Therefore, there is a need to improve reservoir performance and enhance the hydrocarbon recovery mechanism, which is mostly influenced by proper reservoir simulation models. Crichlow (1977) presented a general overview of the simulation approach for the petroleum industry. However, the physical dimension of a reservoir is always an uncertain issue because every reservoir has different geometrical structure and unique geological characteristics (Mustafiz and Islam, 2008). The prediction of reserves based on the theory behind the fluid flow through porous media, and existing mathematical models for the oil displacement process misleading the petroleum industry's existence.

Simulation models capture complex physical phenomena related to the inherent geological complexity of earth models. Due to the nonlinearity complexity in governing equations, the dynamic simulation and solution processes in a petroleum field remains a major challenge and an ongoing research topic. The governing equations for the fluid flow in porous media based on conservation of mass, momentum, and energy equations. Reservoir simulation models categorised into two groups, based on standard black-oil models and compositional models (Aziz and Settari, 1979). In compositional models, conservation equation had written for individual components (Young and Stephenson, 1983). Despite the increase in the use of compositional models, the high computational cost associated with nonlinearity complexities remains a major drawback. On the other hand, the black oil models are more attractive candidates for most reservoir simulation studies in the industry due to their simplifying assumptions regarding realistic field-scale simulations (Lee et al., 2008).

For solving the simulation models, several analytical and numerical methods were applied to handle the nonlinear problems. However, the solutions are not exact due to their possibilities of linearization and various assumptions and failed to provide multiple solutions instead of a single solution for a set of governing equations. Therefore, an advanced numerical tool is needed to predict the exact solution for a multivariable problem and the solutions are realistic rather than impractical.

Based on above issues, the focus of this paper is to review the solution techniques for various nonlinear equations in petroleum reservoir engineering and simulation, technical challenges in solving those governing equations, and future facilitation for the reservoir simulation. Here, we reviewed the related literature of various researcher, summarized their solution techniques and model along with assumptions and properties. Finally, we provide some guidelines for future research and development (R&D).

1.1. Background of the research

In reservoir engineering, most of the equations expressed the nonlinear behaviours due to effects of the time interval, fluid and formation properties (e.g. porosity, permeability, water saturation, viscosity, etc.) variation, distribution of pressure responses, simplification of the governing equations at formulation stage or the feasibility of multiple solutions. Islam and Nandakumar (1986,1990) showed the nonlinear behaviour of the governing equations in petroleum reservoir engineering and simulation. To avoid the nonlinearity, the previous researchers solved the governing equations using linearized methods (i.e. Taylor series expansion, Optimal linearization method, Global linearization method, Perturbation theory, Euler's method, Runge-Kutta method, Newton's Iteration, etc.) along with some assumptions (Jordan, 2006). These procedures help the researcher to understand the simulation model and control the system design methods quickly. However, the linearization effects were significant, and the results were not accurate due to the wrong prediction of the parameters distribution and number of errors. The interpretation of the simulation models also affected by neglecting higher order roots and taking the assumption (Islam et al., 2016). The material balance equation, Navier-Stokes equation, Buckley-Leverett's equation, etc. are the typical example of nonlinear behaviour in petroleum reservoir engineering (Table 1).

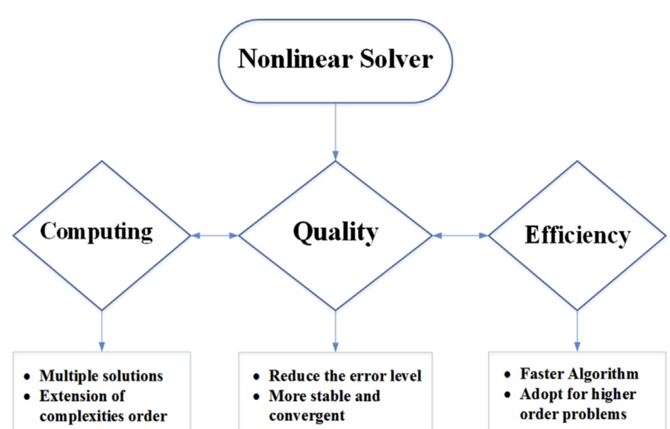

Figure 1. Importance of nonlinear solver.

Table 1
A few examples of nonlinear equations

Sl. No.	Equations	Reason's for nonlinearity	References
01	Material Balance equation	Nonlinear nature of pressure decline with time or distance	Islam et al. (2016)
02	Navier-Stokes equation	Nonlinear stress-rate of strain relationship	Islam et al. (2016)
03	Buckley-Leverett's equation	Nonlinear behaviour due to the inclusion of capillary pressure	Mustafiz et al. (2008b); Islam et al. (2016)
04	Darcy's law: Fluid flow through porous medium	Nonlinear nature of pressure-dependent properties	Abou-Kassem et al. (2006); Islam et al. (2016)