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Recent developments on relationships between the equivalent permeability and fractal dimension of two-dimensional rock fracture networks

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A B S T R A C T

Over the last 30 years, discrete fracture network (DFN) modeling has been increasingly utilized in various practical applications in geoengineering and geosciences. The geometric properties such as the aperture, length, orientation, and connectivity of fractures in the networks significantly influence the permeability of the fractured rock masses. Two key issues include determining the distributions of these geometric properties and establishing relationships between permeability and the geometric properties of DFNs. Previous studies have shown that both single fractures and complex fracture networks exhibit fractal properties, and recent studies have established analytical and/or empirical expressions between the permeability and fractal dimension of the fracture networks. In this study, we review the fractal properties of rock fractures and fracture networks and their correlations with the permeability of DFNs. Analytical, numerical, and analytical-numerical solutions for permeability are individually reviewed, and the simplifications used in each model are extensively discussed. Moreover, ways to determine and improve the mathematical relationships between the permeability and fractal dimension of DFNs in future studies are specifically noted. This work provides a reference for engineers and hydrogeologists who use fractal methods, especially beginners who are interested in predicting the permeability of fractured rock masses.

Contents

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1. Introduction

The permeability of natural fracture networks plays an important role in many geoengineering applications, such as CO_2 sequestration (Juanes et al., 2006, 2010; MacMinn et al., 2010), high-level nuclear waste disposal (Pruess et al., 1990; Cvetkovic et al., 2004), risk assessment of water inrush in karst tunnels and coal mines (Li et al., 2014a, b), and underground crude oil storage facilities (Wang et al., 2015a, b). Since these projects are in the deep underground and the flow velocity is sufficiently small, the fluid flow in natural fractures is commonly governed by the cubic law, in which the flow rate is linearly proportional to the pressure gradient (Zimmerman et al., 2004; Chen et al., 2015a, b). In these tight rock masses, such as granite and basalt, the permeability of fractures is much greater than that of the rock matrix (Murphy et al., 2004; Cai et al., 2010, 2012, Cai and Sun, 2013; Ansari-Rad et al., 2012; Wei et al., 2015; Wang et al., 2016a), and in recent years, the discrete fracture network (DFN) modeling technique has attracted considerable attention with the development of increased computing power (Long et al., 1982; Cacas et al., 1990; Jing, 2003; Erhel et al., 2009). The permeability of a DFN is controlled by its geometric properties (e.g., fracture length, aperture, surface roughness, orientation, density, infilling, and so on) and the applied mechanical and hydraulic boundary conditions (Zhao et al., 2010, 2011, 2014; Liu et al., 2016a). However, the natural fractures inside the underground rock masses are invisible; thus, it is difficult to accurately obtain the real distributions of the fracture features (Bonnet et al., 2001; Parashar and Reeves, 2012). The most commonly used approach is to obtain as much fracture information as possible from the outcrops and well bores to generate DFNs using the Monte Carlo method (Min and Jing, 2003; Min et al., 2004; Baghbanan and Jing, 2007, 2008). Some geometric properties (e.g., fracture length and surface roughness) exhibit fractal characteristics, which have been used to study thermal conductivity (Kou et al., 2009; Xiao et al., 2013; Li et al., 2014c) and spontaneous imbibition (Cai et al., 2014, 2015, 2017).

However, a large number of previous studies characterized the fractal properties of single fractures and fracture networks with no fluid flow involved. Since the permeability of a fractured rock mass is significantly influenced by the geometric properties (e.g., fracture density, surface roughness, connectivity, etc.) of fracture networks correlated with fractal dimension, a correlation should exist between the permeability and fractal dimension of fracture networks. Determining how to use fractal dimension to predict the permeability of a fracture network has been one of the most important issues that many hydrogeologists have studied. Although some analytical and/or empirical models have been proposed, the critical simplifications in these models may yield predicted results that deviate from natural (i.e., actual) cases, and no models have been proposed that can predict the permeability of a fracture network.

In the present study, we aim to review the present studies regarding the correlation between the permeability and fractal dimension of natural fracture networks and extensively note the existing problems that have not been addressed. Section 2 introduces the theoretical background of fractal theory and governing equations of fluid flow in fractures. In Section 3, the previous studies on the fractal dimensions of both single fractures and fracture networks are reviewed. Section 4 presents the fractal-based permeability models in the previous works, including: analytical models (Section 4.1), numerical models (Section 4.2), and analytical-numerical models (Section 4.3). Section 5 discusses the unsolved issues that can facilitate the mathematical relationships between fractal dimension and permeability of fractured rock masses, and lists them as open questions. The present work can help researchers, especially beginners, to understand previous achievements and identify important areas of future research.

2. Theoretical background

2.1. Fractal theory

The fractal theory, as shown in Eq. (1), was first described by Mandelbrot (1982), and this definition can be utilized to characterize the self-similarity of complex patterns (Kruhl, 2013). Additional details regarding fractal theory were given by Fractals, 1988, Falconer (1990), Vicsek (1992), and Bonnet et al. (2001).

$$\mathrm{N}(s) \sim s^{-\mathrm{D}} \tag{1}$$

where N is the number of an object with properties such as length, area, and volume larger than s and D is the fractal dimension of the object.

Many methods have been proposed to calculate the fractal dimension, such as the mass method (Davy et al., 1990; Sornette et al., 1993), box-counting method (Barton and Larsen, 1985; Chiles, 1988), and two-point correlation function method (Walsh and Watterson, 1993; Hamburger et al., 1996; Bour, 1997), among which the box-counting method has been widely used to describe the spatial distributions of fractures with different patterns (Jiang et al., 2006b). Three aspects must be considered to calculate the fractal dimension. First, the fracture region is treated as a square box. Second, by decreasing the size of the square box, the number of boxes that completely cover the fracture-incorporated region can be counted. Finally, a log-log plot of the number of boxes versus the size of square boxes is drawn, and the slope represents the fractal dimension, which is as follows (Falconer, 1990):

$$D = \lim_{\delta \to 0} \frac{\log N_\delta}{-\log \delta} \tag{2}$$

where N_δ is the number of square boxes needed to cover the region and δ is the size of a square box.

2.2. Governing equations of fluid flow in fractures

Fluid flow in rock fractures can commonly be governed by the Navier-Stokes equations, which can be written as follows (Batchelor, 1967; Zimmerman and Bodvarsson, 1996):